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The Laws of Complexity & the Complexity of Laws: The Implications of Computational Complexity Theory for the Law

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THE LAWS OF COMPLEXITY AND THE COMPLEXITY OF LAWS: THE IMPLICATIONS OF COMPUTATIONAL COMPLEXITY THEORY FOR THE LAW

*Eric Kades**

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I. INTRODUCTION & OVERVIEW

Some cases are simply too complicated for judges and juries to manage. This complexity can arise for a wide variety of reasons. As Dickens realized long ago, the law itself can be the source of complexity.¹ In dealing with most of these difficul-

1. See CHARLES DICKENS, BLEAK HOUSE (Norman Page ed., Penguin Books 1971) (1853). The book revolves around *Jarndyce v. Jarndyce*, an interminable proceeding in the Court of Chancery.

ties, there have been no formal tools available either to identify tough cases or to help resolve them. We have simply trusted judges and juries to apply the law as best as they can. Most scholars attempt to tackle legal complexity in the large, taking on a wide variety of factors that contribute to the problem. While such works succeed in raising a wide variety of issues surrounding legal complexity, by their very nature such ambitious agendas offer less in the way of concrete results and solutions.

This article offers a partial remedy to this quandary. Computational complexity theory ("CCT"), a mathematical theory of complexity developed by computer scientists over the last forty years, yields some provable limits to our capacity to find facts and apply legal rules to them.² This article applies CCT's precise definition of complexity to demonstrate some surprising results about seemingly simple laws. On a more general level, it helps to explain a number of broad contours in our legal

2. CCT should not be confused with Chaos Theory, the study of nonlinear equations that exhibit, *inter alia*, large changes in response to small perturbations, and that go through complex, recurring patterns. Recent scholarship has explored the implications of Chaos Theory for a wide variety of legal areas. For the area of securities law, see Lawrence A. Cunningham, *From Random Walks to Chaotic Crashes: The Linear Genealogy of the Efficient Capital Market Hypothesis*, 62 GEO. WASH. L. REV. 546 (1994). For the legislative process, see Vincent Di Lorenzo, *Legislative Chaos: An Exploratory Study*, 12 YALE L. & POLY REV. 425 (1994). For jurisprudence, see Robert E. Scott, *Chaos Theory & the Justice Paradox*, 35 WM. & MARY L. REV. 329 (1993). For administrative law, see J.B. Ruhl, *Complexity Theory as a Paradigm for the Dynamical Law-and-Society System: A Wake-up Call for Legal Reductionalism and the Modern Administrative State*, 45 DUKE L.J. 849 (1996). For the evolution of the legal system, see Mark J. Roe, *Chaos & Evolution in Law & Economics*, 109 HARV. L. REV. 641 (1996); J.B. Ruhl, *The Fitness of Law: Using Complexity Theory to Describe the Evolution of Law & Society and its Practical Meaning for Democracy*, 49 VAND. L. REV. 1407 (1996). For a collection of other cites, see Di Lorenzo, *supra*, at 428 n.20.

This is the first article to apply CCT to legal problems. More generally, CCT has rarely been used outside of the computer science community. Recently economists have begun to draw on CCT to formalize the complexity of some of the systems they model. See Graciela Chichilnisky & Geoffrey Heal, *Catastrophe Futures: Financial Markets and Changing Climate Risks* 8-9 (April 1996) (unpublished manuscript, on file with author).

terrain.

It is important to emphasize that CCT addresses only a subset of the issues surrounding legal complexity. The "return" for this narrower focus is a much deeper understanding of the difficulties when they arise. This is an age-old trade-off: formal, precise methods provide greater understanding than vague intuition, but we can apply them only when all the required assumptions hold.

Part II surveys the literature on legal complexity and concludes by contrasting CCT with existing approaches. In the process of laying out the terrain of legal complexity, this overview offers a sprinkling of comparisons and contrasts between the focus of existing scholarship and CCT. This groundwork is designed to help readers see what sorts of complexity issues CCT does, and does not, address.

Part III provides a layperson's introduction to CCT. Part IV applies the theory to show that specific rules, from mortgage priorities to bankruptcy reorganization classes to the scope of conspiracies, along with some contractual terms, can place effectively impossible demands on any trier of fact. Part V uses CCT to provide novel explanations for, and justifications of, the judiciary's aversion to multi-party disputes, and the existence of private property systems. It concludes by noting some strong parallels between CCT and Lon Fuller's famous article on the limits of adjudication.³ Part VI returns to the more mundane examples of Part III and provides some suggestions for courts facing intractable cases.

II. CONVENTIONAL VIEWS OF LEGAL COMPLEXITY (OR THE COMPLEXITY OF LAWS)

Legal scholars have not had an easy time defining complexity, and some have been disarmingly honest about this difficulty. One author admitted that he was tempted to define complexity by averring, "I know it when I read it."⁴ Commenting

3. Lon Fuller, *The Form & Limits of Adjudication*, 92 HARV. L. REV. 353 (1978).

4. Lance W. Rook, *Laying Down the Law: Canons for Drafting Complex Legislation*, 72 OR. L. REV. 663, 669 (1993). Rook seems to be paraphrasing, *sub silentio*, Justice Stewart's infamous declaration that, while

on the notorious complexity of the Internal Revenue Code, another scholar asserted that, "[n]either 'tax simplification' nor its mirror image, complexity, is a concept that can be easily defined or measured. I know of no comprehensive analytic framework for these ideas, nor any empirical study that supplies a 'simplicity index' in particular areas of tax law practice."⁵

Undeterred by the lack of the sort of "comprehensive analytic framework" that CCT provides for certain types of complexity, scholars have appealed to everyday, intuitive definitions of complexity. Across a wide range of topics, from tax law to environmental law, and from rules in the abstract to litigation in the courtroom, previous work has fleshed out the meaning of legal complexity via laundry lists of "factors" or "sources." While no strong consensus exists on the most important sources of legal complexity, the following section summarizes the most prominent themes that occur in literature. Again, the primary purpose of this survey of literature is to focus on the subset of complexity issues addressed by CCT.

A. *Traditional Types of Legal Complexity*

1. The World Itself

Perhaps the most commonly cited culprit for the complexity of laws and legal systems is the world we live in (and have in large part created).⁶ In societies consisting of millions of citizens with many conflicting aspirations, it is no surprise that legal systems, designed to harmonize disputes, become complex.⁷

Peter Schuck defines *density* as the amount of behavior that

he could not define hard-core pornography, "I know it when I see it." *Jacobellis v. Ohio*, 378 U.S. 184, 197 (1964).

5. Boris Bittker, *Tax Reform & Tax Simplification*, 29 U. MIAMI L. REV. 1, 1 (1974).

6. See Barbara Fishbein, Book Note, 18 HARV. C.R.-C.L. L. REV. 613, 614 (1983) (reviewing JEROLD S. AUERBACH, *JUSTICE WITHOUT LAW?* (1983)).

7. See David M. Schultz, *Market Share Liability in DES Cases: The Unwarranted Erosion of Causation in Fact*, 40 DEPAUL L. REV. 771, 771 (1991) (noting that the certainty for recovery has eroded as the tort law has developed).

a body of law attempts to regulate, and argues that this is a major source of complexity.⁸ To the extent that people in society have intricate relationships, i.e., to the extent the world the law attempts to regulate is complex, so too must the law be complex. Schuck cites pension law as an example of dense laws—rules that are complex because they regulate a myriad of issues between employers, employees, and the government as regulator.⁹

Perhaps an even more compelling example of complexity driven by the intricacy of human affairs is tax law. Since the inception of the federal income tax, commentators have viewed complexity as virtually inevitable.¹⁰ A British expert, reflecting on the complexity of the United States' first income tax statute, smugly noted "[t]here is the usual failure to see that modern life and modern commerce are so complex and diversified that to expect a tax form which shall read like a pill advertisement on the railway, and yet close down upon every case, is asking for the moon."¹¹

Later, leading American tax scholars have echoed this sentiment. "[C]omplexity is in large part a result of the fact that our complicated tax rules are applicable to an enormously complex economic and legal system. Clearly, such a society both engenders and demands a complex tax system."¹² "It is unfortunately the fact that, by its very nature, a tax on income must take account of an almost infinite spectrum of business, investment, and personal events and transactions Income taxation entails a high level of irreducible complexity."¹³

8. See Peter H. Schuck, *Legal Complexity: Some Causes, Consequences, and Cures*, 42 DUKE L.J. 1, 3 (1992).

9. See *id.* at 3-4.

10. See John A. Miller, *Indeterminacy, Complexity, and Fairness: Justifying Rule Simplification in the Law of Taxation*, 68 WASH. L. REV. 1, 7 n.25 (1993) (discussing the intricacy and the degree of complexity involved in federal income tax law).

11. Robert B. Eichholz, *Should the Federal Tax Be Simplified?*, 48 YALE L.J. 1200, 1212 (1939) (citing Dr. J.C. Stamp, Testimony before the British Royal Commission on the Income Tax (August 1, 1919), in *Cmd. 1920*, 615 Minutes of Evidence, I, ¶ 9481).

12. Stanley S. Surrey, *Complexity and the Internal Revenue Code: The Problem of the Management of Tax Detail*, 34 LAW & CONTEMP. PROB. 673, 686 (1969).

13. Bittker, *supra* note 5, at 2. More recent works on the complexity

One reply to this justification for the tax laws' complexity is that the opportunities for technological improvements in the administration of, and compliance with, the tax system should be at least as great as the opportunities for technological improvements in other areas of the economy. "The tax system should enjoy net efficiency gains like the gains enjoyed by the [rest] of the economy."¹⁴ Indeed, tax law seems precisely like the kind of formal, mechanical set of rules for which computers are ideally suited, to manage more and more complexity with fewer and fewer (human) headaches. CCT shows, however, that this intuition is not always correct.¹⁵

Scholars in other areas have, like tax experts, argued that the complexity of the real world "entails a high level of irreducible complexity" in the law. Eric Orts has criticized contractarian and law and economics approaches to corporate law because they ignore "the complex nature of the relationships about which corporate law is concerned."¹⁶ "[R]ecent attempts to define or formulate a unified theory of 'the corporation' fail to account for the complexity of corporate law."¹⁷ He argued that "[f]orcing the [complex] world into rigid theoretical boxes is dangerous,"¹⁸ and like Bittker, feels that corporate law is irreducibly complex. Alyson Flournoy similarly argues that "the ever-increasing complexity that characterizes our relationship to the environment"¹⁹ has undermined simple cost-benefit analysis of environmental problems.

of the Internal Revenue Code continue to make the same point: "Whatever tax law we adopt must apply to a large, multi-faceted world that has generated intricate economic arrangements." Edward J. McCaffery, *The Holy Grail of Tax Simplification*, WIS. L. REV. 1267, 1275 (1990).

14. Hal Gann & Roy Strowd, *The Enormous Complexity of Being Fair*, 66 TAX NOTES 1711, 1711 (1995).

15. See *infra* Part III.

16. Eric W. Orts, *The Complexity & Legitimacy of Corporate Law*, 50 WASH. & LEE L. REV. 1565, 1575 (1993).

17. *Id.* at 1566.

18. *Id.* at 1623.

19. Alyson C. Flournoy, *Coping with Complexity*, 27 LOY. L.A. L. REV. 809, 809 (1994).

2. Multifaceted Notions of Justice

As these critics imply, laws could be much simpler, even given the complexity of human affairs, if society strived for fewer and simpler ideals. If, for instance, we decided that all laws should aim only to increase economic output in the short run, environmental law would become much simpler (or perhaps disappear). There are, of course, objections to creating laws that serve such simple goals. As a practical matter, notions of justice and fairness are also important determinants of legal rules. They compound the legal complexity caused by the real world.

Even seemingly simple conceptions of fairness quickly lead to more complex laws. Tax law, for instance, uses such seemingly straightforward notions as *horizontal equity*: similar treatment of taxpayers in similar situations.²⁰ Yet when legislators and courts deal with a myriad of situations in the real world, they must add layer after layer of rules to preserve even the simplest types of fairness. "Much of our [tax law] statutory detail arises from the feeling that this pressure for fairness should—in fairness—be satisfied."²¹

Of course, most notions of fairness and justice are not nearly as simple as horizontal equity. More intricate definitions create what John Miller, in discussing tax law, calls judgmental complexity: "the intellectual, moral, and philosophical burdens a tax question may pose."²² He finds that "[m]ost often judgmental complexity arises because more than one rule or principle may apply to a given taxable event, and those potentially applicable principles are in conflict."²³ Multiple and conflicting

20. For a traditional definition of horizontal equity, see RICHARD A. MUSGRAVE & PEGGY B. MUSGRAVE, *PUBLIC FINANCE IN THEORY AND PRACTICE* 223 (5th ed. 1989). Recent scholarship has questioned whether horizontal equity has any content beyond vertical equity, the principle that those with higher incomes should pay more taxes. See generally Louis Kaplow, *Horizontal Equity: Measures in Search of a Principle*, 42 NAT'L TAX J. 139 (1989) [hereinafter Kaplow, *Horizontal Equity*]; see also Louis Kaplow, *A Note on Horizontal Equity*, 1 FLA. TAX REV. 191 (1992) [hereinafter Kaplow, *Note on Horizontal Equity*].

21. Surrey, *supra* note 12, at 700.

22. Miller, *supra* note 10, at 12.

23. *Id.* For a similar observation in a different area of the law, see

goals translate into more complex rules.

Ronald Dworkin has vividly described the difficulty of trying to harmonize complex and divergent ideas of justice. It takes a Hercules of a judge, he maintains, to construct "coherent" laws that form a "seamless web" that will bring justice in line with a society's ideals.²⁴ Dworkin argues that it is a judge's duty to strive for the Herculean ideal regardless of the difficulty (or impossibility) of the task. For Dworkin, as for these other scholars, complex laws are the inevitable price of multi-dimensional definitions of justice.

3. Indeterminacy

Without rules to resolve conflict between different principles of justice, a legal system is facially inconsistent—it is *overdetermined* and thus there are no "right" outcomes in many cases. This seems unacceptable. Ideally, rules designed to control conflict between the different aspects of justice lead to a unique solution in every case. This, however, is quite difficult to achieve.

Instead, legal rules often appear indeterminate. They admit of more than one solution. Schuck argues that indeterminate rules lead to complexity because they are "open-textured, flexible, multi-factored, and fluid [T]urning on diverse mixtures of fact and policy, indeterminate rules tend to be costly to apply and their outcomes are often hard to predict."²⁵ Schuck cites the reasonableness standard in tort law as one example of an indeterminate rule that introduces complexity into the legal system.²⁶

McCaffery notes that foundational terms in the tax laws are not tightly defined. "For example, for the traditional income tax, these will involve questions of measurement, attribution, and timing."²⁷ Measurement of income sounds simple enough, but case law reveals the unending dispute over what does and

Orts, *supra* note 16, at 1566-67 ("Corporate law is complex because it embodies a set of conflicting normative principles.").

24. RONALD DWORKIN, *LAW'S EMPIRE* 239-40, 264, 354, 379-91 (1986).

25. Schuck, *supra* note 8, at 4.

26. *See id.* at 11.

27. McCaffery, *supra* note 13, at 1274.

what does not fall under the rubric of "income."²⁸ Congress left the law in a highly indeterminate state when it specified the base of the income tax as "all income from whatever source derived."²⁹

Schuck notes that the usual cure for indeterminacy, more rules, may itself introduce further complexity.³⁰ McCaffery labels this "dynamic complexity,"³¹ an iterative battle between taxpayers finding loopholes under an indeterminate revenue code, and the IRS continually attempting to plug those loopholes.³² Reductions in complexity via reduced indeterminacy seem to be offset by greater technical complexity.

Miller argues that indeterminacy, and the complexity it engenders, is not necessarily a bad thing. Detailed rules are more deterministic, but Miller argues that they end up being arbitrary or unjust in many cases.³³ More general principles, though indeterminate, can seem more fair. This is nothing more than the age-old tension between rule of law (determinate, but potentially heartless) and rule of men (less determinate law varying from judge to judge, but hopefully suffused with flexible standards of justice). As previously intimated,³⁴ justice, by requiring indeterminacy, may make complex legal rules inevitable.

4. Cognitive Difficulties

The previous sections have focused on complexity stemming from the intricacies of the real world, the cross-currents in widely held notions of justice, and the ambiguities inherent in

28. For two of the more famous cases on the subtleties in fleshing the seemingly simple definition of "gross income" (defined as "all income from whatever source derived," 26 U.S.C. § 61(a) (1994)), see *Eisner v. Macomber*, 252 U.S. 189 (1920) (holding that a stock dividend on common stock is not gross income). For a further discussion, see also *Commissioner v. Glenshaw Glass Co.*, 348 U.S. 426 (1955) (holding that punitive damages awards are included in gross income).

29. 26 U.S.C. § 61(a).

30. See Schuck, *supra* note 8, at 4.

31. McCaffery, *supra* note 13, at 1275-76.

32. See *id.*

33. See Miller, *supra* note 10, at 22.

34. See *supra* Part II.A.2.

indeterminate rules. Cognitive issues focus more closely on the relationship between rules, and the human minds that produce them. Analogizing it to arithmetic, here are two ways of saying the same thing:

$$2 = 2$$

$$177,991 - 88,427 + 901 - 109,280 + 18,817 = \int_0^1 x \, dx + (\sin(45^\circ))^2 \log_2 \sqrt{2} + \frac{1}{2}(\sin^2 x + \cos^2 x)$$

The first expression is verifiable on sight; the second requires some work. It is easy to recognize the validity of the first, but the second poses greater cognitive difficulties.

Ideally, legal rules should be comprehensible on a quick read by a nonspecialist.³⁵ In practice, many statutes (and common law rules) are what Schuck calls *technical*.³⁶ They are intricate and require specialized skill to manage efficiently. The tax code is his prototypical example of technical rules.³⁷ McCaffery, using the same label, defines technical as "the pure intellectual difficulty of ascertaining the meaning of tax law."³⁸ Although he uses a different label, Miller cites the same problem with tax law. "[E]laborative complexity relates to the level of information and education that must be absorbed in order to begin to decide a tax question. Thus, the length and detail of tax rules, along with their interconnectedness, are directly related to their elaborative complexity."³⁹ Miller agrees that "[t]he problems associated with elaborative complexity are a function of human frailty."⁴⁰ Technical (elaborative) rules present cognitive difficulties for people. In discussing what makes some statutes difficult to parse, Rook identifies three common attributes that people find vexing:

35. This perspective sounds innocuous and most scholars discussing legal complexity seem to accept it, yet it could well have served as the motto of iconoclastic Legal Realists earlier in this century. See WILLIAM TWINING, KARL LLEWELLYN AND THE REALIST MOVEMENT 70-73 (1973) (discussing Legal Realism in the 1930s).

36. See Schuck, *supra* note 8, at 4.

37. See *id.*

38. McCaffery, *supra* note 13, at 1274.

39. Miller, *supra* note 10, at 12.

40. *Id.* at 12.

- (1) exceptions within exceptions within exceptions . . . ;
- (2) verbal representations of mathematical relationships; and
- (3) cross-references to other rules.⁴¹

Empirical work indicates that cognitive problems figure prominently in the difficulty people have with laws. Statistical results from a survey of tax experts indicate that (i) excessive detail of statutory and regulatory provisions, and (ii) requirements of numerous calculations, together, explain most of what experts label as complex rules.⁴² CCT, as we shall see, deals with aspects of complexity entirely removed from such cognitive shortcomings of human beings.

B. Litigation Complexity

So far this article has focused on literature analyzing the complexity of legal rules in and of themselves, outside of additional complications caused by actual litigation. CCT applies only to such "pure" rule complexity. In order to clarify the issues addressed by this article, however, it is useful to briefly discuss work on the complexity of litigation. The article then returns to rule complexity and summarizes the main normative explanations for it.

While the complexity of litigation is not entirely a modern phenomenon,⁴³ modern interest was triggered in large part by the difficulties the federal courts were facing with the increasingly larger antitrust suits that arose after World War II. The first systematic effort to prescribe techniques for judges (and to a lesser extent lawyers) to deal with such cases defined complex litigation in highly functional terms: "cases which present unusual problems and . . . require extraordinary treatment, including but not limited to the cases described as 'protracted' or 'big.'"⁴⁴ The definition in the third version of this treatise has changed little: "the need for [judicial] management . . . is

41. Rook, *supra* note 4, at 669-70. Note that attempting to design a systematic procedure to check for certain types of cross-referencing problems, such as circular referencing, is futile. See *infra* Part III.A.

42. See Susan B. Long & Judyth A. Swingen, *An Approach to the Measurement of Tax Law Complexity*, 8 J. AM. TAX ASS'N 22, 32 tbl. 7.

43. See, e.g., DICKENS, *supra* note 1.

44. MANUAL FOR COMPLEX LITIGATION 3 (3d ed. 1995) (quoting the first edition definition of complex litigation) [hereinafter THE MANUAL].

[the] defining characteristic: The greater the need for management, the more 'complex' is the litigation."⁴⁵

In discussing the complexity of different substantive areas of the law, The Manual seems to explain complex litigation in terms of (i) the complexity of the real world (that the law allows to enter as evidence),⁴⁶ and (ii) cognitive difficulties for judges, lawyers, and juries.⁴⁷ For instance, The Manual attributes the complexity of antitrust cases to "voluminous documentary and testimonial evidence, extensive discovery, complicated legal, factual, and technical (particularly economic) questions"⁴⁸ The focus on volume of evidence seems to indicate that antitrust law does not abstract from the complexity of real-world economic relationships. Parties can submit evidence on a wide range of issues, and the judge and jury are left to sort out the factual mess. The Manual's discussion of complicated and technical questions of law and fact seem to refer to cognitive difficulties faced by judges and juries. The same is true for The Manual's description of complexity in patent cases. "The principal source of complexity in patent litigation generally is the technical nature of the subject matter. Its unfamiliarity poses unique problems for judges and juries."⁴⁹

45. *Id.* For a similar functional definition, see Jay Tidmarsh, *Unattainable Justice: The Form of Complex Litigation and the Limits of Judicial Power*, 60 GEO. WASH. L. REV. 1683, 1691 n.21 (1992) ("my search for a definition of 'complex litigation' is limited to the context of procedural reform: What meaning should we give to 'complex litigation' if we are going to create special rules to handle it?"). Both The Manual and Tidmarsh focus on the judge's role, and to a lesser extent, the lawyer's role in complex litigation. For a discussion of the jury's role, see generally Roger W. Kirst, *The Jury's Historic Domain in Complex Cases*, 58 WASH. L. REV. 1 (1982) (advocating use of the judge-jury historical test in ascertaining the role of juries in complex litigation).

46. See discussion *supra* Part II.A.1.

47. See discussion *supra* Part II.A.4.

48. THE MANUAL, *supra* note 44, § 33.1.

49. *Id.* § 33.6. Jay Tidmarsh, *supra* note 45, at 1757-58, 1766, also discusses real-world complexity and cognitive difficulties as significant sources of complexity in litigation. He labels lawyers' difficulty in formulating simple legal theories *formulational* complexity, and argues that it is, in large part, a product of informational overload. See *id.* at 1757-58. He labels factfinders' difficulty in weighing evidence *decisionmaking* complexity and argues that the "factfinder may not have the intellectual

Another complicating factor The Manual mentions repeatedly is multiplicity of parties and disputes. "[L]itigation involving many parties in numerous related cases . . . requires management and is complex"⁵⁰ In mass tort cases, for instance, which often involve multiple plaintiffs and defendants, "[p]ronounced conflicts may exist among the defendants, and the filing of third-party complaints may result in the joinder of numerous additional parties."⁵¹ Jay Tidmarsh also cites joinder as a major source of complexity.⁵²

Lon Fuller articulated a more focused view of complexity in the courts due to the number of parties or, more generally, the number of interacting "forces."⁵³ He argued that courts only deal well with bipolar conflicts. Courts are not equipped to handle "polycentric" policy disputes that involve multiple interests.⁵⁴ While this article will not further discuss other issues of litigation complexity, Part VI.B does apply CCT to provide a theoretical foundation buttressing Fuller's perceptive insights.

C. Normative Analyses of Complexity

While The Manual offers a battery of tools and tricks for coping with involved cases, it offers no systematic theory for weighing the costs of complexity against its benefits (if any). While no other works fill in this gap for litigation complexity, a number of scholars, applying law and economics, have developed cost-benefit analyses of the complexity of individual legal rules.

Not all authors in this area would necessarily agree that

capability to understand" either the facts or how the law applies to them. *Id.* at 1766. In the cognitive vein, he also notes that the "factfinder's life experiences may be so foreign to the key issues in the case that the factfinder cannot make the factual or legal inferences required by the case." *Id.* at 1766.

50. THE MANUAL, *supra* note 44, § 10.1.

51. *Id.* § 33.2.

52. See Tidmarsh, *supra* note 45, at 1714 (advocating the court's role in denying joinder to avoid complexity).

53. See Fuller, *supra* note 3, at 394-99 (arguing that polycentric disputes, involving multiple parties, cannot effectively be adjudicated by a judge acting in his traditional role).

54. See *id.*

they are analyzing complexity. The seminal work in this area considered "the benefits and costs associated with different choices along the continuum between the highly specific rule and the highly general standard"⁵⁵ The authors label this "precision," and carefully note that length of a rule may be a poor proxy for it:

Theoretically, the precision of a given law can be measured by the number of elementary situations or circumstances that are identified by that law to be either included in or excluded from the universe of circumstances to which a sanction applies. Thus, precision refers to the information content of a law rather than to the number of provisions included in a given law.⁵⁶

A more recent work applies the label "complexity" to what sounds like the same concept: "the complexity of legal rules refers to the number and difficulty of distinctions the rules make . . . the more difficult it is to determine the applicable category—whether the difficulty involves understanding the rules themselves or ascertaining the relevant facts—the greater complexity is said to be."⁵⁷

Dean Diver's work,⁵⁸ avowedly a discussion of precision,⁵⁹ advances the analysis, suggesting that there is significant overlap between these economic studies and complexity as viewed by the authors previously discussed.⁶⁰ He breaks precision down into three components:

- (1) *transparency*: "use words with well-defined and universally accepted meanings";
- (2) *accessibility*: "applicable to concrete situations without excessive difficulty or effort"; and

55. Isaac Ehrlich & Richard A. Posner, *An Economic Analysis of Legal Rulemaking*, 3 J. LEGAL STUD. 257, 261 (1974).

56. *Id.* at 281.

57. Louis Kaplow, *A Model of the Optimal Complexity of Legal Rules*, 11 J.L. ECON. & ORG. 150, 150 (1994).

58. Colin S. Diver, *The Optimal Precision of Administrative Rules*, 93 YALE L.J. 65 (1983).

59. *See id.* at 66 ("Before we can begin to make useful prescriptions about the precision of administrative rules, we must give the concept some added precision of its own.").

60. *See* discussions *supra* Part II.A, II.B.

(3) *congruence*: "whether the substantive content of the message communicated in his words produces the desired behavior."⁶¹

While congruence does not fit neatly into the factors contributing to legal complexity discussed above, transparency and accessibility seem to involve real-world complexity and cognitive difficulties.⁶²

Cognitive difficulties, at bottom, explain the costs of precision/complexity described by these authors. Posner and Ehrlich focus on the expense of formulating rules that must categorize diverse behavior precisely, and note that such rules may require citizens to hire (expensive) experts (e.g., tax lawyers) to help them avoid liability.⁶³ Building on the latter point, Kaplow emphasizes the costs of determining the application of more complex rules, and notes that to the extent people decide the costs exceed the benefits, the rules will not alter behavior.⁶⁴

While most of the literature summarized above focuses only on the costs of legal complexity,⁶⁵ one of the major contributions of the economic approach is to shed light on the potential offsetting benefits. Posner and Ehrlich divide these benefits into two categories. First, and perhaps of greater potential importance,

[a] perfectly detailed and comprehensive set of rules brings society nearer to its desired allocation of resources by discouraging socially undesirable activities and encouraging socially desirable ones. This is because detailing the law efficiently . . . results in an increase in the expected gains from engaging in socially desirable activity relative to that from engaging in undesirable activity.⁶⁶

61. Diver, *supra* note 58, at 67.

62. *See id.*

63. *See* Ehrlich & Posner, *supra* note 55, at 267-71.

64. *See* Kaplow, *Horizontal Equity*, *supra* note 20, at 151, 153-55 (stating that if the cost of complying with the rule is prohibitive, the rule will simply be ignored).

65. Schuck's work is an exception; he explicitly analyzes legal complexity in terms of its costs versus its benefits. *See* Schuck, *supra* note 8, at 7. Schuck focuses on sociological factors that may produce inefficiently complex laws. *See id.*

66. Ehrlich & Posner, *supra* note 55, at 262. The adverb "efficiently"

For example, vague laws on libel may chill socially desirable speech; the costs of complexity in cases like this seem to be outweighed by the benefits of greater specificity. To the extent more detailed (complex) rules enable society to alter behavior in socially beneficial ways, complexity, far from evil, is positively desirable.

Second, Ehrlich and Posner argue that precision yields a number of benefits within the legal system. Greater detail will produce fewer violations of the law (those due to uncertainty); prosecutors can deploy resources in a more focused way (more efficiently); greater predictability will encourage settlement;⁶⁷ and information about disputes will be cheaper to gather and communicate given the tight focus provided by detailed laws.⁶⁸

Once lawmakers list all the relevant costs and benefits of complexity, optimizing the amount of complexity in the law becomes a straightforward exercise in economic marginalism. One continues to add incremental precision to a rule as long as the sum of the benefits of the new twist exceed the sum of its costs. As in most areas, outlining this law and economics theory is much easier than applying it in the real world (where

is important here; it means that the drafter makes no errors that inadvertently deter desirable conduct or fail to deter undesirable conduct. See *id.* at 268-70.

67. See RICHARD A. POSNER, *ECONOMIC ANALYSIS OF LAW* § 21.5 (4th ed. 1992) (arguing that if litigants can easily quantify the respective probability of winning in litigation [as a result of more predictable legislation], he or she can more easily quantify his or her gain or loss as a result of litigation and are more likely to settle as a result).

68. See Ehrlich & Posner, *supra* note 55, at 264-67. Those factors listed in the text do not exhaust the authors' discussion. Diver consciously follows Ehrlich & Posner's cost-benefit analysis. See Diver, *supra* note 58, at 73-74 & n.36. While Kaplow extends their analysis significantly on the cost side from formulation to application of legal rules, he does not extend their analysis of the benefits of complexity. See Kaplow, *supra* note 57, at 150. In another article, Kaplow further departs from the Ehrlich & Posner approach by emphasizing that, in addition to level of detail, a major difference between general standards and specific rules is that rules decide most cases *ex ante*, while standards postpone decisions until actually presented in litigation. Louis Kaplow, *Rules versus Standards: An Economic Analysis*, 42 DUKE L.J. 557 (1992).

lawmakers must obtain accurate estimates of the abstract costs and benefits delineated in the literature). That said, this approach does seem to provide a useful framework for thinking about the complexity of specific legal rules and groups of rules.⁶⁹

Richard Epstein has taken this normative perspective to its limit, *defining* legal complexity in terms of the cost of obedience. "[T]he cheaper the cost of compliance, the simpler we can say the rule is" ⁷⁰ To illustrate, he notes that while most lawyers (and especially law students) think the Rule Against Perpetuities is among the most complex of rules, they and their clients can avoid its pitfalls by adding a simple savings clause specifying how to distribute assets if the rest of the instrument contains a perpetuities violation. In contrast, the tax laws pose true, unavoidable complexity since there is no such "escape hatch." Based on these insights, Epstein argues that "the *minimum* condition for calling any rule complex is that it creates public regulatory obstacles to the achievement of some private objective."⁷¹

Epstein realizes that complex rules may yield the benefits discussed earlier in this section: better incentives for aligning narrow private incentives with the wider public good. Given a perceived present-day "bias toward overregulation," however, Epstein believes that "the presumption should be set in favor of a simplification of legal rules."⁷²

Applying this normative framework, Epstein categorically rejects the notion discussed above,⁷³ that an increasingly complex world requires increasingly complex tax laws. "As a normative matter, the conventional view of the subject has matters exactly backward. The proper response to more complex societies should be an ever greater reliance on simple legal

69. For a recent and somewhat quirky and informal examination of this topic by a prominent economist who cites none of the literature discussed in this section, see Gordon Tullock, *On the Desirable Degree of Detail in the Law*, 2 EUR. J.L. & ECON. 199 (1995).

70. RICHARD ALLEN EPSTEIN, *SIMPLE RULES FOR A COMPLEX WORLD* 25 (1995).

71. *Id.* at 27 (emphasis in original).

72. *Id.* at 36.

73. See discussion *supra* Part II.A.1.

rules"⁷⁴ Laws everywhere must deal with the same problems, scarcity of resources and human selfishness, and the complexity of society does not alter the best solutions, which Epstein argues remain simple rules.

This same scarcity of resources, combined with the straightforward observation that, like any other good in a world of scarcity, "justice is subject to the law of diminishing return,"⁷⁵ leads Epstein to caution against investing excessive resources in the complex, multifaceted notions of justice, previously discussed.⁷⁶ He pragmatically warns against aiming for unachievable perfection:

The only question for the legal system is how it will make its errors, not whether it will make them. Simple rules are adopted by people who acknowledge that possibility up front, and then seek to minimize it in practice. Complex rules are for those who have an unattainable vision of perfection.⁷⁷

D. How Computational Complexity Theory Differs from These Approaches: An Overview

Epstein then addresses complexity in the large, analyzing its society-wide impact. "Legal complexity is not merely a measure of the inherent or formal properties of legal rules. It is also a function of how deeply they cut into the fabric of ordinary life."⁷⁸ Epstein examines specific rules (such as the Rule Against Perpetuities) to gauge their costs and benefits, and implicitly, like the law and economics approach summarized in the preceding section, studies costs and benefits *as the complexity/precision of rules increases*. CCT concentrates on a different dimension of complexity. CCT takes the rules as fixed, and analyzes the difficulty of applying them *as the size of the case to which they must be applied increases*. To take an example from a topic we shall later discuss at length,⁷⁹ priori-

74. EPSTEIN, *supra* note 70, at 21.

75. *Id.* at 38.

76. See discussion *supra* Part II.A.2.

77. EPSTEIN, *supra* note 70, at 39.

78. *Id.* at 29.

79. See discussion *infra* Part IV.A.1.

ties among creditors, the law and economics approach⁸⁰ would ask, "What are the costs and benefits of adding a rule to deal with cases of circular priorities?" This paper, applying CCT, instead asks, "How much more difficult is it to detect and deal with circular priorities when the number of creditors increases from 3 to 10? to 25? to 100?"

While no previous work has explicitly weighed the difficulty of applying rules in the face of larger and larger cases, such questions seem just as important in practice as the costs and benefits of adding precision to existing rules. Moreover, while the costs and benefits in law and economics analysis are virtually impossible to gauge in practice, CCT provides precise measurements of complexity. Admittedly, CCT achieves this precision by considering solely what Epstein labels the "formal properties of legal rules," and cannot address "how deeply [rules] cut into the fabric of ordinary life" across as broad a swath of issues as his and others' analyses. To reiterate, CCT sacrifices breadth for depth in understanding one type of legal complexity.

It is easy to underestimate how quickly the size of a problem can make a set of rules practically useless. Although this article explains the problem in detail,⁸¹ note for now that simply because a jury or a computer can solve a problem involving three creditors in three minutes in no way implies that they can solve a 100-creditor problem in 100 minutes. CCT shows that, depending on the rules, it may take more or less time—sometimes as much time as has elapsed since the birth of our universe in the Big Bang.⁸²

It is also quite difficult for laymen (and often experts) to spot those sets of rules for which time requirements increase rapidly with the size of the problem. Consider a collection of cities, with some pairs connected directly by a road, while other pairs require travel through at least one other city. It is relatively easy to find a path that traverses each stretch of road exactly once, or to show that no such path exists, even as the number of cities and roads increases to very high numbers.

80. See discussion *supra* Part II.C.

81. See discussion *infra* Part III.C.

82. See DAVID HAREL, ALGORITHMICS 156 fig.7.3 (2d ed. 1992) ("the number of microseconds since the 'Big Bang' has 24 digits").

On the other hand, finding a path that visits each city exactly once quickly becomes infeasible as the number of cities increases above about twenty.⁸³

All the results of CCT hold for computers as well as humans. While many people believe that computers can perform any mechanical calculations relatively quickly, the main result of CCT is that for some problems this is not true. The fact that CCT applies to computers as well as humans tells us that it reveals a type of complexity that has nothing to do with cognitive difficulties.⁸⁴ Computers have no difficulty dealing with the intricate rules that may be difficult for humans. As Rook has noted, multi-level exceptions within exceptions are trivial for computers, as are cross-references between rules.⁸⁵ It is true that computers generally cannot deal with verbal or written representations of mathematical relationships, but once such problems are "translated" into computer programs (their native tongue), computers are right at home with math problems.

CCT does address the complexity of the real world and of complex notions of justice. To the extent that complexity in the real world translates into bigger problems, to which rules must be applied in court (e.g., greater number of creditors), CCT helps us gauge precisely how quickly complexity will overwhelm factfinders. To the extent that multifaceted notions of justice lead us to draft involved rules, CCT tells us when our desire for justice crosses the threshold from practicality to impracticality.

In addition to "narrow" applications that show when particu-

83. See *id.*, at 128. For illustrations of how quickly this task becomes unmanageable, see *infra* Part III.E. CCT's version of complexity, as these simply-stated examples show, has nothing to do with the length of a law. Ehrlich and Posner disavow length as a proxy for complexity, but they appeal to an ill-defined term, "information content," which for decently drafted laws (those without meaningless clauses) would seem highly correlated with length. See Ehrlich & Posner, *supra* note 55, at 281. Other legal complexity scholars have recognized that at least some forms of complexity have nothing to do with the length of a law. See Miller, *supra* note 10, at 7 ("[e]ven a complex thought can sometimes be stated briefly.").

84. See discussion *supra* Part II.A.4.

85. See discussion *supra* Part II.A.4.

lar legal rules are unworkable for cases of significant size, CCT provides insights into more general issues of what is legally complex and why. After explaining CCT and using it to show that some existing legal rules are indeed impractical in bigger cases, the article will conclude by examining these broader applications.

III. A PRIMER ON COMPUTATIONAL COMPLEXITY THEORY (OR, THE LAWS OF COMPLEXITY)

Users of computer programs require two things: that programs give correct answers (i.e., do what they claim to do), and that the programs yield such correct answers in a reasonable amount of time. The first requirement is undoubtedly more important, since an incorrect answer is worth little, if anything, no matter how fast a computer produces it. Unfortunately, theorists have been able to provide only minimal assistance to programmers who wish to *prove*, as a matter of logical certainty, the correctness of their software.⁸⁶

Theorists have, however, developed a substantial body of results to help programmers both to specifically gauge the time it will take their programs to run, and more generally to assist them in writing programs that run efficiently. This body of knowledge is CCT. Its main aims are (i) to assess how quickly programs will run, and (ii) to prove that a given approach is the fastest way to attack a problem. The following section provides readers with enough background in CCT to understand its relevance to legal rules.

A. *Truly Impossible Tasks (meta-tasks, in a sense)*

Before analyzing the complexity of a task and programs that solve it, theorists have another important contribution to offer: certain tasks are provably impossible. These tasks fall outside the ambit of complexity theory, into another computer science subfield called the theory of computation. Perhaps the most famous result of the theory of computation is that it is impossi-

86. See, e.g., NISSIM FRANCEZ, PROGRAM VERIFICATION (1992); JACQUES LOECKX & KURT SIEBER, THE FOUNDATIONS OF PROGRAM VERIFICATION (2d ed. 1987).

ble to write a computer program that takes any and all other computer programs as input and determines whether or not they will halt on any possible input.⁸⁷ This would be an incredibly useful program, since, with such a tool, programmers could insure that their programs would never go into infinite loops (i.e., word processors would never get "hung" and become unresponsive to any user input). While it might be possible to construct a program that would detect the possibility of failure to halt in "most" cases, it is impossible to predict the cases for which this programmer's assistant would fail. Theorists have thus saved programmers the wasted effort of trying to construct the unconstructable. Computer programmers of course strive continuously to write programs that will not "hang," and there are tools that help them detect some such cases. In the end, however, there is no general-purpose way to eliminate the possibility of programs running forever on some input.

The theory of computation does have some lessons for legal theorists and legislators. Analogize a society's entire set of legal rules to one big computer program, and the facts of a particular case to a program's input. If a legal system permits rules to make reference to other rules and contains conditional statements (if x do A, otherwise do B), we have the possibility of precisely the kind of conditional "jumps" in "control" that can lead a computer program to cycle endlessly (hang). Our statutes are filled with such cross-references and conditionals, and it is possible that buried in the statute books is a *cycle* of

87. For an elementary and lucid proof of this important result, see HAREL, *supra* note 82, at 206-09. The idea of a computer program that takes other computer programs as input may strike the non-expert as bizarre. Such programs are, however, common and are increasingly important in modern software development. Programmers use "debuggers" to help find out where their programs produce errors, and they use performance analyzers to detect where the program spends the most time. Even more important are the compilers that translate computer languages in which humans can efficiently write computer programs (e.g., BASIC, Fortran, C) into the 1's and 0's that microprocessors can deal with; compilers are themselves programs that take as their only input other programs. See THEODORE GYLE & J.W. DOERR, MINICOMPUTERS STRUCTURE & PROGRAMMING 180-81 (1976) (defining "computer fundamental #4," which states that compilers accelerate the programming steps needed to design and implement programs).

references such that, when applied to some fact pattern (input), leads lawyers through an endless, repetitious circuit of rules. The theory of computation tells us that it is futile to attempt to create an automatic procedure to detect such an unseemly possibility that will work in every case. The cost of such certainty would be to sacrifice cross-referencing, a cost we have apparently decided is not worth paying, perhaps under the belief that such an endless cycle of cross-references is highly unlikely.⁸⁸

B. Classifying the Complexity of Possible Tasks

The remainder of this article assumes that the tasks facing programmers (and later, judges) are feasible, and focuses on the question of how long it takes proposed solutions to perform them. Just as legislators and judges do not craft different rules for different sized cases (e.g., the rules for resolving priorities among creditors apply whether there are three or three hundred of them), so too computer programmers do not write different programs to, e.g., sort lists of ten names and lists of 100 names. In both cases, authors create solutions that can take input varying in size from nothing to instances of arbitrarily large size.

In the realm of computers, theorists estimate the running time of a program by considering the number of elementary steps (e.g., discrete actions by the central processor) needed to solve a problem. As previously touched upon,⁸⁹ the time it takes to complete a task, i.e., the number of elementary steps, of course depends on the size of the input—basically, the size of the specific instance to which the program is being applied.

A couple of examples familiar to many lawyers should help clarify this point. First, the amount of time it takes to find all cases that contain a word or phrase on LEXIS or Westlaw⁹⁰

88. For the application of some close relatives of the theory of computation to jurisprudential questions, see generally John M. Farago, *Intractable Cases: The Role of Uncertainty in the Concept of the Law*, 55 N.Y.U. L. REV. 195 (1980); John M. Rogers & Robert E. Molzon, *Some Lessons About the Law from Self-Referential Problems in Mathematics*, 90 MICH. L. REV. 992 (1992).

89. See *supra* Part II.D.

90. LEXIS is a trademark of the Reed Elsevier Plc Group; Westlaw is

depends on the size of the database being searched.⁹¹ It will take longer to search for all cases from every American court that has ever used the phrase "circular priorities" than it will to search for the same phrase in only North Dakota cases decided in the last five years. There is much more ground to cover in the first instance, and the computer will have to march through many more elementary operations (such as comparing text, character by character). This translates into a longer running time.

Another example that will be familiar to lawyers who use word processors is construction of a table of authorities at the beginning of a brief. The longer the brief, and the greater the number of citations, the longer it will take a computer to process all the relevant information and spit out the table.

CCT formalizes these insights to produce quantifiable measures of how long a program will take to process input of various sizes. One way to present such information is in a table. For instance, LEXIS or Westlaw might provide customers with the following summary:

Number of Cases Being Searched	Time to Complete Search (in microseconds)
100	1,001
1,000	10,001
10,000	100,001
100,000	1,000,001

This provides a rough idea of the relationship between the size of the problem and the time it will take to do it, but such a table is incomplete. Users might ask about the time necessary

a trademark of West Publishing Corporation.

91. To be technically precise, the size of the word or phrase a user is searching for also affects the time (number of steps) a search takes, but this is a minor factor compared to the size of the database being searched. See, e.g., GYLE & DOERR, *supra* note 87, at 220-21 (explaining that the average time of a database search is directly proportional to the length of the database). For further discussion about database searches, see *id.* at 210-36.

if there are seven cases, or seventeen, or 17 million. We could build a huge table with every number from one to the total number of cases in existence, but that is unmanageable. There is a better way.

CCT describes the time it takes to perform a task as a *function* of the size of the input. These are the same functions that algebra teachers have tortured high school students with for years. Putting the table above into functional form, we have one example of a time complexity function:

$$\text{Time-1} = 10 \times (\# \text{ of cases}) + 1$$

Users now have no need for a gargantuan table; they can take the number of cases they need to search, plug it into this function, and directly obtain the time their search will take.

The casual observer may focus on the two numbers in the function, 10 and 1, in assessing the general performance of the search program. In CCT, however, these numbers (the first is called a coefficient, the second a constant) are of little consequence. This is easiest to understand for constants like the "1" at the end of the function. Since it makes an absolutely fixed contribution to the time the program takes, no matter how many cases the program searches, its contribution to the total time (in percentage terms) becomes minuscule rather quickly. This is somewhat analogous to the observation that fixed costs do not affect economic decisions at the margin.

Understanding why coefficients (like the "10" in the function Time-1) do not matter takes a bit more thought. After all, since this factor significantly increases the running time as the number of cases grows, it would seem to be crucial to CCT. To see why such coefficients are relatively unimportant in measuring the growth rate of our Time function, consider a proposed new program to search through cases. After examining it, we determine that its running time function, Time-2, is defined as follows:

$$\text{Time-2} = .002 \times (\# \text{ of cases}) \times (\# \text{ of cases})$$

This new program's time function has a much smaller coefficient (5000 times smaller) than Time-1, and it has no constant factor at all. There is, however, the somewhat troubling circumstance that the function multiplies the number of cases by itself. Rewriting Time-2 in more compact notation, we express this self-product with an exponent:

$$\text{Time-2} = .002 \times (\# \text{ of cases})^2$$

We are interested in comparing how fast Time-1 and Time-2 increase as the number of cases increases—i.e., the growth rate of the time functions. Let's build a table to compare the growth rates of Time-1, with its higher coefficient (and constant), and Time-2 (with its squared term). In addition to calculating both run times, the table includes a final column expressing Time-2 as a percentage of Time-1.

Number of Cases Being Searched	Time-1 Completion	Time-2 Completion	Time-2 as % of Time-1
100	1,001	20	2%
1,000	10,001	2,000	20%
10,000	100,001	200,000	200%
100,000	1,000,001	20,000,000	2000%

While the lower coefficient (and, to a much smaller extent, the lack of a constant term) cause Time-2 to have a faster running time for searches involving up to 1000 cases, somewhere in the interval between 1,000 and 10,000 cases Time-1 becomes clearly preferable. Moreover, in percentage terms Time-1's advantage increases as the number of cases increases. This example shows that, in analyzing the growth rate of functions, exponents on the variable(s) (here, the single variable is the number of cases) are the determinative factor "in the long run." The effect of such self-multiplication of the variable, here the number of cases, overpowers constants and coefficients. Specifically, this example shows that a quadratic function (like Time-2, with a variable squared) will eventually exceed a linear function (like Time-1, with no exponents on the variables).⁹²

92. The relative unimportance of coefficients and constants answers questions that may have occurred to some readers. Does the running time of a program depend on the specific computer it is running on? Does this make it impossible to discuss running time in general? The answer to the first question is "yes." The difference in running time between computers can vary by a factor of ten, a thousand, a million, or even more. Note, however, that whatever the difference, it is a fixed

Comparisons between functions like Time-1 and Time-2 are the essence of CCT. If given the choice between a linear and a quadratic program that perform the same task, CCT says that, in general, the linear program is preferable since it will do the job in less time.⁹³ Roughly speaking, CCT views all linear programs that solve the same problem as "equals," or more precisely, as members of the same complexity class (all quadratic functions are members of a less preferable class). That is not to say that all linear running time solutions to the same problem are equal; lower coefficients lead to quicker performance and so are, of course, preferred. The percentage difference between two linear programs, however, does not change with the size of the input. Unlike the quadratic example, Time-2, users do not pay an ever-increasing penalty (in percent terms), as the size of the input increases, when using an inferior linear-program.

Since Time-1 is preferable to Time-2, should programmers stop there, or should they look for an even better solution? This brings us to the second major goal of CCT: helping programmers determine the *fastest possible* way to solve a problem. In technical terms, this is called establishing a *lower bound* on the complexity class of a problem. For instance, it is impossible to search cases for a word in less than linear time. To see why this is true, note that any faster algorithm must necessarily fail to examine at least some words, for any algorithm that examines each word is by definition linear (under the assumption that examining a word takes a fixed number of elementary steps). Establishing lower bounds for problems is more difficult than assessing the complexity of a given program, but it is no less important.

multiple. All programs, roughly speaking, will run a *fixed percent* faster on one machine than on any given slower machine. Since coefficients are irrelevant to CCT, the theory applies across all computers. A program that is linear on IBM's first PC is linear on the fastest supercomputer today. It will run much faster on the latter, but the advantage of the supercomputer, again, in percent terms, will not increase as the size of the input grows.

93. It is possible to imagine situations where this is not true. If (i) the coefficient (and the constant) in a quadratic function are much smaller, and (ii) it is known that the size of the input will never be very large, then a quadratic running time program will be preferable to a linear one. Such examples, however, are rare in practice.

*C. The Line Between Tractable and Intractable Problems:
Exponentials v. Polynomials*

In most cases, a linear algorithm is the best we can hope for. Finding a linear procedure is very good news to computer scientists.⁹⁴ We saw in the previous section that the running time of quadratic programs (those with an exponent of two on a variable) are usually longer. The difference gets worse as the exponent of the variable increases; thus cubic programs (exponent of three) take longer than quartic programs, and quartic programs (exponent of four) take longer still.

Now add a strange twist: programs whose time complexity function has the input size variable *as an exponent*. For instance, we can imagine a case searching algorithm with the following running time function:

$$\text{Time-3} = 2^{(\# \text{ of words})}$$

One way to think of such functions, aptly called exponentials, is as weird polynomials where the *degree* of the polynomial increases with the size of the variable. Thus, for instance, this exponential searching algorithm looks like a quadratic function for documents with two words, a cubic function for documents

94. One exception of a program that runs even faster than linear is a search of a sorted list to see if the given entry is contained. By first examining the middle item, and then comparing it to the item the user is searching for and concentrating on that half of the list where the entry may lie, a program can find any entry in much fewer guesses than the total number of items in the list. Specifically, such a program can locate an item (or show its absence) in, at most, the logarithm of the number of entries. For example, we can find an entry in a list of 10 in at most four guesses, an entry in a list of 100 in at most eight guesses, and an entry in a list of 1000 in at most 12 guesses. See GYLE & DOERR, *supra* note 87, at 224 (explaining the mathematical basis for the search algorithm).

It is also possible to obtain what appears to be faster than linear programs by precomputing part of the solution before users run a program. For instance, both LEXIS and Westlaw compile indexes of almost all of the words in their databases so that when users perform a search, the program looks in a sorted list to see if a case has a given word, instead of marching through the (unsorted) case itself. Electronic Mail from John P. Hourigan, Lexis-Nexis Corporation, to the author (July 18, 1996). The reason this is not really better than linear, for the purposes of CCT, is that assembling the indexes in the first place takes more than linear time.

with three words, and so on.

The growth rate of exponential functions is striking.⁹⁵ The following tables help illustrate the difference between linear, various polynomial, and various exponential running times.⁹⁶

Func- tion	N (Think of N as number of words to be searched)				
	10	50	100	300	1000
5N	50	250	500	1500	5000
N ²	100	2500	10,000	90,000	1 x 10 ⁶
N ³	1000	1.25x10 ⁵	1 x 10 ⁶	27 x 10 ⁶	1 x 10 ¹⁰
2 ^N	1024	1 x 10 ¹⁶	1 x 10 ³¹	1 x 10 ⁹¹	1 x 10 ³⁰²
N ^N	1 x 10 ¹⁰	1 x 10 ⁸⁵	1 x 10 ²⁰¹	1 x 10 ⁷⁴⁴	unimag- inably large

Perhaps surprisingly, most programs with polynomial time complexity functions run fast enough that they can work in

95. Some lawyers have long been aware of this fact. See *Armstrong v. McGhee* 1795 Add. 261 (County Ct. Westmoreland, Pa., 1975) (attorney arguing that a contract that called for a price as "a barley corn for the first nail, in his shoes, and so in a duplicate ratio for every other," would not be "fair [or] reasonable"). By "duplicate ratio," I believe the lawyer meant that the number of grains would double with each nail. This is an example of an exponentially growing quantity.

At times legal scholars have blurred the important distinction between polynomials and exponentials. See POSNER, *supra* note 67, at 51 (noting that "generally the costs of a transaction rise with the number of parties to the transaction—perhaps exponentially. The formula for the number of links required to join all members of a set of n members is suggestive in this connection: $n(n-1)/2$."). *Id.* The formula, however, is quadratic (2nd order polynomial), not exponential. As discussed *infra* Part IV.B, the quantity that grows exponentially with the number of parties is the number of possible coalitions (of varying size) that the individuals might form.)

96. HAREL, *supra* note 82, at 166 Fig. 7.3.

practice even on problems of relatively large size.⁹⁷ Exponential functions, however, are on the other side of the divide. The rate of increase is out of control. When the amount of time it takes to solve a task grows exponentially with the size of the input, we say that the task is intractable—it is solvable in theory, but would take an eternity for problems of even modest size, as the table above illustrates.

[E]xponential algorithms should not be considered 'good' algorithms, and indeed this usually is the case. Most exponential algorithms are merely variations on exhaustive search, whereas polynomial time algorithms generally are made possible only through the gain of some deeper insight into the structure of the problem. There is wide agreement that a problem has not been 'well-solved' until a polynomial time algorithm is known for it. Hence, we shall refer to a problem as intractable if it is so hard that no polynomial time algorithm can possibly solve it.⁹⁸

The line separating polynomials from exponentials is the line

97. This is true in practice because most programs that run in polynomial time are of relatively low degree, typically quadratic or cubic. A program with a running time function of degree, ten thousand, admittedly, would not be very useful. It would be worse than an exponential running time program for all problems of size up to (roughly) ten thousand.

98. MICHAEL R. GAREY & DAVID S. JOHNSON, *COMPUTERS AND INTRACTABILITY* 8 (1979). If the size of the problem is guaranteed to fall below a given level, say 10, then an exponential program might work faster than a polynomial of a degree higher than 10. Also, on rare occasion a program that has an exponential running time ends up working well in practice. This illustrates a subtle but important aspect of CCT ignored in the text: most commonly, computer scientists use a worst-case analysis of a program's running time. It is possible for a program to work very quickly for most inputs, but very slowly on a small class of examples. Such examples, however, seem to be the exception rather than the rule. There are two additional reasons computer scientists use worst-case analysis instead of a seemingly more appealing average-case analysis. First, as a matter of mathematics, it is often difficult or impossible to calculate average-case running time. Second, worst-case calculations avoid surprises. They provide programmers with an absolute bound on the running time of their programs; users might be surprised by algorithms with a good average-case running time, if they apply the program to a problem that takes much longer than average.

separating problems that can be solved by programs that run in a reasonable time from those that cannot.⁹⁹

Note that even the now predictable leaps in computer performance, with speeds doubling about every eighteen months,¹⁰⁰ are of only marginal help with problems that require exponential time. Such doubling of machine speed doubles the size of problems we can solve with linear programs. Even for a quadratic running time program it increases the size of solvable problems by around forty percent. For exponential problems, doubling computer speeds increases the scale of solvable problems by only one unit. It takes a very long time, at that rate, to get to problems of serious size.

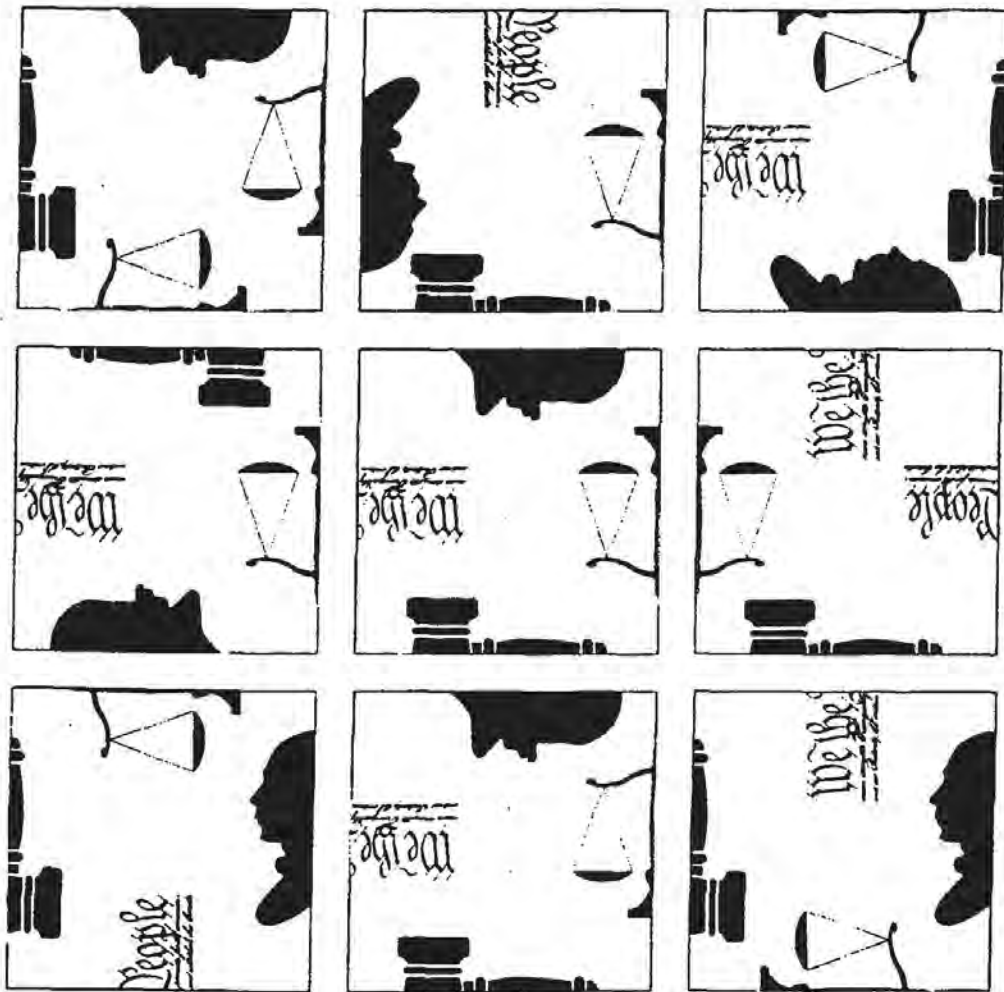
D. Provably Intractable Tasks

In order to understand what computer scientists mean when they state that "exponential algorithms should not be considered 'good' algorithms" because they "are merely variations on exhaustive search," consider the following legal variant of a well-known puzzle. The goal is to fit the nine squares together

99. It is important to note that this distinction between polynomials and exponentials is not the relevant distinction in other contexts. For instance, Dean Clark argues persuasively that businesses organize hierarchically in order to limit the number of channels of communication, and the costs that such communication entails. See ROBERT C. CLARK, *CORPORATE LAW* app. A, 801-16 (1986). In a completely democratic firm, on the other hand, where any employee could talk with any other employee, the costs of communication would undoubtedly be higher. See *id.* at 805-06. As a first estimate, however, the increase appears akin to that of moving from a linear program to a quadratic one in the following sense. In a strict hierarchy, each employee can only talk with one person "above" her, and so the number of possible communication links is equal to the number of employees (i.e., linear in the number of employees). See *id.* at 807-08. Under complete democracy, the number of such links rises only to a function that is quadratic in the number of employees. See Kaplow, *supra* note 64. If Dean Clark is correct, a polynomial number of communication links imposes inefficient costs, even before we contemplate an exponential number of such links.

100. "Moore's Law (named after Intel cofounder Gordon Moore) . . . states that the number of transistors on a chip will double every 18 months or so. Performance has increased at nearly the same rate." Michael J. Miller, *Microprocessors March On*, P.C. MAG., Dec. 17, 1996, at 4.

into a three-by-three square so that all of the internal edges match. It may help to motivate readers if they photocopy the page, cut out the pieces and try to solve the puzzle.



Those who become frustrated have good company. There is a solution, but to date no one has produced a systematic way of finding it. The only sure way to solve it is to try every possible arrangement of the nine pieces. This is what is meant by "exhaustive search," and even for seemingly small problems it is exhausting. At first blush that may not sound onerous, but the number of possible arrangements for this innocuous-looking three-by-three puzzle is over 95 billion.¹⁰¹ Even for the trivial-looking two-by-two version of the puzzle there are 6,144 possible arrangements. For a four-by-four puzzle we again see the power of exponentiation: the number of arrangements goes from the billions to a 22-digit number, roughly equaling the number of microseconds (millionth of a second) since the Big Bang.¹⁰²

It is easy to construct computer programs that take an exponential amount of time. CCT is more concerned with identifying those tasks that cannot be solved any more quickly; i.e., for what problems is an exponential running-time program the best possible solution? Such problems are intractable, as defined above.

The answer consists of good news and what all theorists suspect is bad news. First the good news: there are very few tasks that we *know without a doubt* take an exponential amount of time. The canonical example of a task that takes exponential time is listing all the subsets of a given set. This takes an exponential amount of time since the number of subsets increases exponentially as we add elements to a set.¹⁰³

101. This number is derived as follows. First, there are 9! (the "!" means factorial: $9! = 9 \times 8 \times 7 \times \dots \times 1$) ways to place the nine pieces (upper left, upper center, . . . , lower right). Once we have placed them, there are four ways to rotate each piece, and this amounts to 4^9 total rotational arrangements for each of the 9! placement patterns. Taking the product, we have

$$9! \times 4^9 = 95,126,814,720$$

potential solutions.

102. See HAREL, *supra* note 82, at 166. Here are the calculations for the two-by-two and the four-by-four puzzles:

2 by 2: $4! \times 4^4 = 6144$;

4 by 4: $16! \times 4^{16} \approx 8.986 \times 10^{22}$

(\approx means "approximately;" this number has been truncated).

103. To take a concrete example of this type of task, consider in the

Most other examples are closely related to this task.¹⁰⁴

E. Probably Intractable Tasks

The suspected bad news is that many common problems *probably* take an exponential amount of time to solve. These problems comprise a complexity class with the esoteric name of NP-complete ("NP-c"),¹⁰⁵ that has defied decades of efforts to find efficient (polynomial) solutions. They can all be solved in exponential time with exhaustive search but, as we have seen, that is really no solution at all. It has become a virtual holy grail of CCT to prove that these problems do indeed require an exponential number of steps—that this is the lower bound on the time required for any solution to them. This writer, like almost all computer scientists, works on the assumption that

context of business organizations and lines of communication between employees, *see* Clark, *supra* note 99, and the number of subcommittees. If we define a subcommittee as any group of employees that share mutual lines of communication (all have access to each other), then in the completely democratic firm the number of subcommittees grows exponentially as the number of employees increases. In a hierarchical firm, on the other hand, the number of such committees grows in only a linear fashion.

104. The number of moves required to solve some puzzles and to find a winning strategy to a number of well-known games turns out to increase exponentially as the size of the game increases. The Towers of Hanoi puzzle, for instance, requires players to move a set of disks of decreasing size from one pole to another with the use of a third pole for temporary storage. The rub is that players cannot place a larger disk on top of a smaller one. As the number of disks increases, the number of moves required to accomplish the task increases exponentially. *See* HAREL, *supra* note 82, at 161-62. While CCT does not apply to traditional chess since the game is of a fixed size, finding a guaranteed winning strategy for *generalized* chess, where the rules allow for the addition of pieces and increases in the size of the board, takes an exponentially increasing amount of time. *See* Aviezer S. Fraenkel & David L. Lichtenstein, *Computing Perfect Strategy for $n \times n$ Chess Requires Time Exponential in n* , 31 J. COMBINATORIAL THEORY 199 (1981).

105. This means "nondeterministic polynomial complete." Without getting into technical details irrelevant for the purposes of this article, solutions to all these problems can be verified in a polynomial number of steps. GAREY & JOHNSON, *supra* note 98, at 28 ("It is this notion of polynomial time 'verifiability' that the class NP is intended to isolate."). Finding the solution in the first place, however, is the rub.

NP-c problems are computationally intractable.

An incredibly broad range of problems have been proven to be in the NP-c category, and thus likely require exponential time to solve.¹⁰⁶ This article has already discussed one of the most famous examples:¹⁰⁷ trying to find a path on a network of roads that starts and ends in a city and that passes through every other city exactly once. This problem, with intractable variants, comes up in a wide variety of contexts, from establishing airline schedules to routing telephone calls. It is quite easy for computers (and humans) to solve examples with five or six cities.

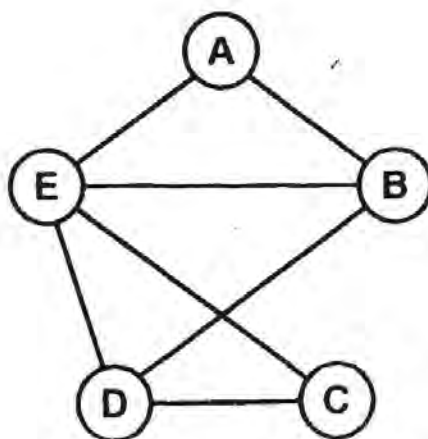


Figure 1

One solution to this simple example is A, B, D, C, E, A. The problem becomes difficult when the number of cities reaches ten.

106. See GAREY & JOHNSON, *supra* note 98, app.

107. See *supra* Part II.D.

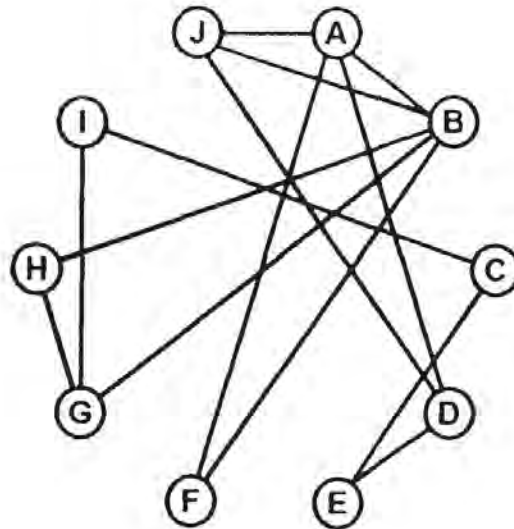


Figure 2

One solution to this example is A, F, B, H, G, I, C, E, D, J, A. When the number of cities reaches twenty, the problem becomes intractable.¹⁰⁸

108. See GAREY & JOHNSON, *supra* note 98, at 199.

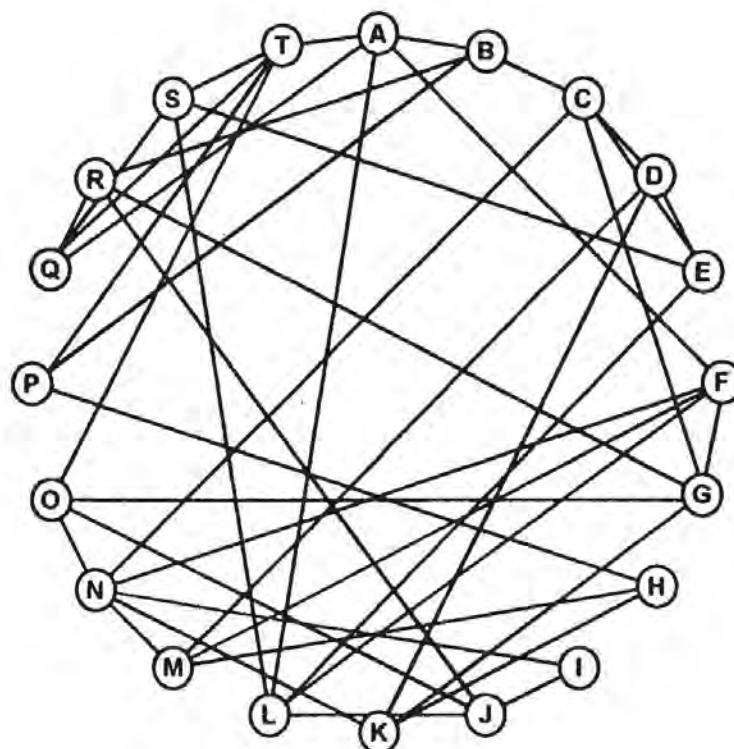


Figure 3

The author does not know if there is a solution; such examples are much easier to create than to solve. Such networks (or graphs), involving vertices (cities) and edges (roads), are a rich source of NP-c problems.¹⁰⁹

Here is another colorful example. Given a knapsack of fixed volume and a collection of objects with fixed volume and worth,

109. See *id.* at 190-220.

maximize the value placed in the sack.¹¹⁰ Again, when the number of objects is relatively small, computers and humans can find the solution easily.

Knapsack Volume: 20

Item #	Volume	Value
1	10	50
2	8	40
3	7	35
4	6	27
5	5	24
6	4	20

As the size of the problem becomes even modestly large, finding a solution is extremely difficult.

Knapsack Volume: 20

Item #	Volume	Value
1	20	60
2	19	55
3	18	54
4	17	51
5	12	39
6	11	35
7	10	29
8	9	25
9	5	14
10	4	12
11	3	10
12	2	8

110. See *id.* at 247.

Part IV, *infra*, explores a number of additional examples of intractable problems.

F. The Computer Science "Answer": Heuristics

When faced with intractable problems, programmers cannot simply throw their hands up. Airlines need schedules, phone companies need calls routed, and contractors need timetables. While programmers know that they cannot provide optimal solutions, they experiment with techniques to provide the best feasible solution.¹¹¹ Programs that take this approach ("I know it's not the best answer, but it's the best I can do") are called *heuristics*.

There are a number of tricks that may help programmers craft heuristics. If most or all examples faced in practice are some special case of the more general problem, it may turn out that the special case is tractable. Or, for some problems, a program may work quite well in practice despite the fact that there are a few examples that cause it to run for an inordinately long time.¹¹²

In some cases, CCT can help programmers with theoretical results. For some NP-c problems, theorists can prove that a certain approach always comes within some predictable percentage of the optimal solution. The bad news continues, however. Other NP-c problems do not guarantee even approximate solutions.¹¹³ In general, coming up with heuristic solutions for NP-c problems is as much an art as it is a science.

111. To be more precise, economics predicts that programmers will continue to look for better solutions as long as the marginal benefits, in terms of a better solution, exceed the marginal costs, i.e., programmer's time, computer resources, etc.

112. For example, the simplex method for linear programming (optimizing a linear function subject to linear constraints), while having an exponential worst-case running time, usually solves problems relatively quickly—indeed more quickly than the first polynomial method discovered. See HAREL, *supra* note 82, at 188. A subsequent polynomial method, however, has proved markedly faster. See Neal Karmarkar, *A New Polynomial-Time Algorithm for Linear Programming*, 4 COMBINATORICA 373 (1984).

113. See HAREL, *supra* note 82, at 182-83.

G. Comparing Humans to Computers

Before applying CCT to legal rules, readers may wonder what pertinence, if any, a definition of complexity taken from computer science has for the law. In order to understand CCT's relevance to legal rules, it is useful to compare and contrast the abilities of computers and humans. The following table lists representative tasks with which computers and humans have either an easy or a difficult time handling.

		COMPUTERS	
		Easy	Hard
H U	Easy	<ul style="list-style-type: none"> • simple arithmetic 	<ul style="list-style-type: none"> • recognizing faces • philosophizing • creativity (legal, artistic, . . .)
M A N S	Hard	<ul style="list-style-type: none"> • involved arithmetic • exceptions within exceptions with in . . . • searching large amounts of text 	<ul style="list-style-type: none"> • intractable problems

We do not use computers for problems that humans find easy (nobody uses a calculator to add two plus two), but they have become indispensable for tasks involving huge amounts of arithmetic and more complex math. Computers crunch numbers much more rapidly and accurately than humans. In the same vein, computers have made impracticable chores routine (e.g., searching 10,000 cases for the word "circular" within a thousand words of "path").

There remains, however, a large domain of problems for which human abilities far outstrip computers. Despite periodic hype from artificial intelligence experts, computers have shown no signs of creativity.¹¹⁴ Their ability to perform some seem-

114. See DEREK PARTRIDGE & JON ROWE, *COMPUTERS AND CREATIVITY* 114 (1994).

ingly mechanical tasks, such as recognizing faces, while improving, still lags far behind that of humans.¹¹⁵

This article focuses on the final category of intractable tasks that are difficult for both computers and humans. CCT embodies a notion of complexity independent of both human cognitive weaknesses and raw computational power. While advances in education can reduce cognitive difficulties, and advances in computer technology can make some previously unsolvable problems manageable, intractable problems are stubbornly immune to such "attacks."¹¹⁶ Since such problems will never be fully solved, the law, like other disciplines, must decide how to deal with them when presented, or how to avoid facing them in the first place.

What is potentially deceptive about these problems is that, to those unfamiliar with CCT, they sound like the kind of mechanical tasks for which computers can provide solutions when human capabilities become overwhelmed. This may lead lawmakers to adopt rules based on the false belief that computers can help if a particular case becomes too involved. The next section shows that some existing rules contain the potential to unmask such wishful thinking.

115. See generally MONIQUE PAVEL, *FUNDAMENTALS OF PATTERN RECOGNITION* (2d ed. rev. and expanded 1993).

116. This is why Gann & Strowd's argument, that technology should offset the increasing complexity of the real world must be qualified. See GANN & STROWD, *supra* note 14, at 1711. To the extent a more complicated world generates bigger problems that are NP-c, gains in technology (e.g., the speed of computers) are nowhere near enough to cope. See *supra* text accompanying note 14.

IV. ACTUAL CASES OF FACTUALLY INTRACTABLE TASKS IN THE COURTS

A. *Cycles, Cycles Everywhere*

Rules invariably attempt to reshape chaos into *order*, and one of the most natural ways of imposing order is an *ordering* of items. For example, land recording statutes attempt to place mortgages and other interests in order based on the time an interest holder records, and, at times, on knowledge of other (unrecorded) claims. UCC Article 9 aims to place secured personal property creditors within a similar framework.¹¹⁷

As the subsections below show, however, sometimes a strange thing happens on the way to forming well-ordered queues: a cycle occurs. Party A comes ahead of Party B; Party B comes ahead of Party C; and Party C comes ahead of Party A. Graphically:

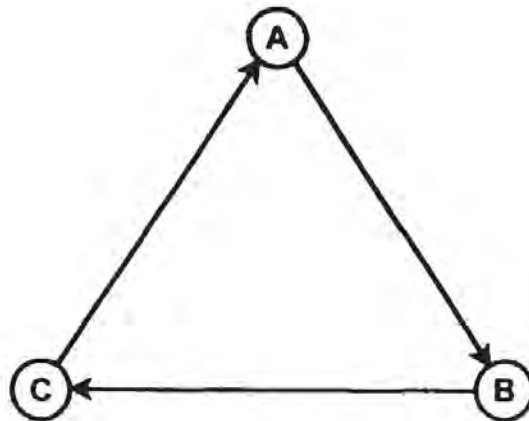


Figure 4

This section will show the wide variety of ways in which this anomaly can arise.

117. See U.C.C. § 9-312(5) (1995); M. Stuart Sutherland, Note, *Circular Priority Systems Within the Uniform Commercial Code*, 61 TEX. L. REV. 517, 518 (1982).

Courts and scholars have long¹¹⁸ recognized this possibility. The phenomenon has been almost universally lamented; typical are declarations that circular priorities are "insoluble on any known principles."¹¹⁹ Grant Gilmore wryly commented, "[a] judge who finds himself face to face with a circular priority system typically reacts in the manner of a bull who has been goaded by the picadors: he paws the ground and roars with rage. The spectator can only sympathize with judge and bull."¹²⁰

Circular priorities continue to pose, at a minimum, "a minor embarrassment" to legal rulemakers.¹²¹ Despite many efforts to deal with cycles, many quite subtle,¹²² there is still no uniform approach. "Any solution [to circular priority problems] may seem arbitrary and produce anomalous results."¹²³

No previous work, however, has focused on an even more difficult antecedent issue, that of detecting cycles in the first place. While parties and courts may stumble across them in particular cases, it turns out that, in general, systematically finding cycles in relationships between a number of parties is NP-c.¹²⁴ This means, to summarize the previous section, that as the number of parties in a case grows to even moderate size, the difficulty of dealing with cycles is dwarfed by the difficulty

118. According to one author, the first circular priorities case is over three hundred years old. See Note, *Circuity of Liens—A Proposed Solution*, 38 COLUM. L. REV. 1267, 1267 (1938) (citing *Greswold v. Marsham*, 22 Eng. Rep. 898 (Ch. 1685)).

119. *Andrus v. Burke*, 48 A. 228, 229 (N.J. Eq. 1901).

120. 2 GRANT GILMORE, *SECURITY INTERESTS IN PERSONAL PROPERTY* § 39.1 at 1020-21 (1965). Gilmore remains the authoritative treatment on circular priorities. For an earlier version of substantially the same material, see Grant Gilmore, *Circular Priority Systems*, 71 YALE L.J. 53 (1961).

121. See Michael D. White, *Untangling Circular Priorities Under UCC Section 9-114*, 86 COM. L.J. 231, 233 (1981).

122. See 2 GILMORE, *supra* note 120, § 39.2.

123. *In re Holly Knitwear, Inc.*, 356 A.2d 405, 409 (N.J. Super. Ct. App. Div. 1976).

124. See GAREY & JOHNSON, *supra* note 98, at 213. To be precise, it has been shown that determining whether there is a cycle of a given size is an intractable task. See *id.* From this it follows that a number of related problems are also intractable: finding the longest cycle, as well as finding all cycles.

of detecting them in the first place.

1. Circular Priorities Among Creditors and Claimants

a. Subordination

Consider a borrower who has granted a first mortgage on Blackacre to A, a second mortgage to B, and a third mortgage to C. In return for some valuable consideration, A then waives his priority over C in what is often called a subordination agreement. We now have the following set of priorities:

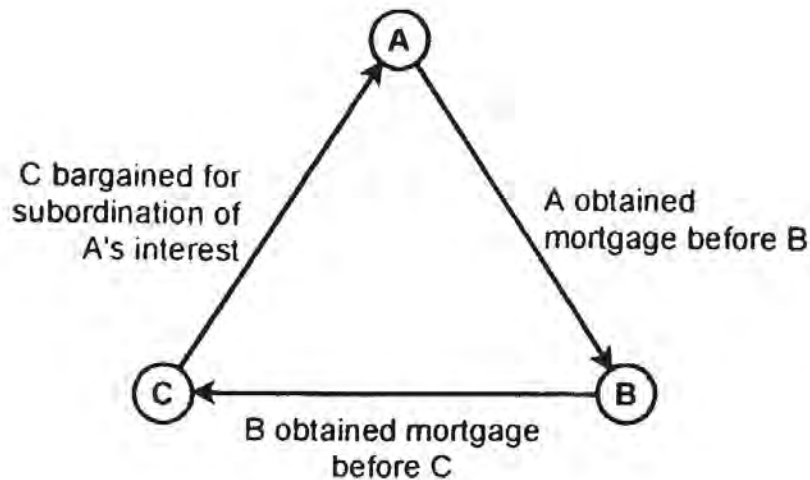


Figure 5

Two of the three parties have, by agreement, created a circular system of priorities. If the borrower defaults, there is no ordering left to control payments to the three creditors.

In this case of circularity created by a subordination agreement, at least, "[t]here is a comforting unanimity, among courts and commentators, on the proper distribution of the fund."¹²⁵ Dubbed the subordination rule by Gilmore, it calls for payment according to the following rules:

1. Set aside from the fund the amount of A's claim.

125. 2 GILMORE, *supra* note 120, § 39.1, at 1020.

2. Pay the amount so set aside to
 - a) C, to the amount of his claim;
 - b) A, to the extent of any balance remaining after C's claim is satisfied.
3. Pay B the amount of the fund remaining after A's claim has been set aside.
4. If any balance remains in the fund after A's claim has been set aside and B's claim has been satisfied, distribute the balance to
 - a) C,
 - b) A.¹²⁶

This rule effectuates the intent of the agreement between A and C, without affecting the amount received by B (who is not a party to the subordination agreement).¹²⁷ Nevertheless, courts continue to apply this rule to solve circular priorities created by subordination agreements.¹²⁸

126. *Id.*

127. The solution is an illustration of the more general *nemo dat qui non habet* rule: "[h]e who hath not cannot give." BLACK'S LAW DICTIONARY 1037 (6th ed. 1990). Applied to this example, A cannot give C what A does not have. In the context of this example, A can grant C priority over B only to the extent that A has such priority.

128. See *In re Cliff's Ridge Skiing Corp.*, 123 B.R. 753 (Bankr. W.D. Mich. 1991) (applying the subordination rule to determine priority of sale proceeds). For an unusual example, see *United States v. Gila Valley Irrigation Dist.*, 920 F. Supp. 1444 (D. Ariz. 1996). In this case, the Gila River Indian Community (GRIC) had prior appropriation rights for Gila River waters dating from time immemorial. The Apache Tribe had prior appropriation rights dating from 1846, held in trust for the tribe by the United States, and the Upper Valley Defendants (UVDs) had prior appropriation rights dating from 1872. Under the terms of a 1935 consent decree, however, the UVDs received apportionment rights that in some cases trumped the GRIC's priority. See *United States v. Gila Valley Irrigation Dist.*, 31 F.3d 1428, 1431 (9th Cir. 1994). The district court explicitly found that there was a circular priority problem, see *Gila Valley*, 31 F.3d at 1457, and followed the subordination rule laid out. See *supra* Part IV.A.1.a; *Gila Valley*, 31 F.3d at 1459-60. For a more complete history of the case, see *United States v. Gila Valley Irrigation Dist.*, 454 F.2d 219 (9th Cir. 1972) and *Gila Valley Irrigation Dist. v. United States*, 118 F.2d 507 (9th Cir. 1941).

b. Recording Systems

Because the circularity created by subordination agreements is consensual, it makes sense to resolve it based on the intent (and power) of the parties. Not all circular priority situations, however, are this simple. Almost any recording system, for instance, can give rise to circular priorities without subordination and without any other agreement among creditors.

The basic idea of any recording system is that a creditor or claimant's place in line is based on the order of filing — first in time, first in right.¹²⁹ Pure race systems, however, where time of filing is *all* that matters, are quite rare.¹³⁰ For real property, most states use either a notice or a race/notice recording system.¹³¹ The key notion in both is that of the Bona Fide Purchaser (BFP). A BFP is a buyer who (i) has no knowledge of any other unrecorded interests, and (ii) pays fair value. BFPs are deemed to be on notice of all recorded interests.¹³² In a notice system, the rule is that the latest BFP has the superior claim;¹³³ in a race/notice system, the rule is that if there are ever multiple BFPs, the first of them to record has the superior claim.¹³⁴

Circularities can arise in both systems because creditors may know of earlier interests that have not yet been recorded. For example, assume A takes a first mortgage on Blackacre to secure a loan, but fails to file. B, *with actual knowledge of A's interest*, takes a second mortgage, and records. C, *without any knowledge of A's unrecorded interest*, takes a third mortgage and records. Under either a notice or a race/notice system, we then get the following example of a circular priority:

129. See 6A RICHARD R. POWELL & PATRICK J. ROHAN, POWELL ON REAL PROPERTY § 82.01[1][a] (1996).

130. Only North Carolina and Louisiana have pure race recording acts. See *id.* § 82.02[1][c][i] n.8. Article 9 of the UCC creates a recording system that is much closer to pure race than most real estate recording acts, but, as shown later in this section, even it has small exceptions that can lead to circular priorities. Note that in a truly pure race system circular priorities can arise only by subordination agreements.

131. See 6A POWELL & ROHAN, *supra* note 129, at § 82.01[2][b].

132. See *id.* § 82.01[1][b].

133. See *id.* § 82.02 [1][c][iii].

134. See *id.* § 82.02[1][c][iii].

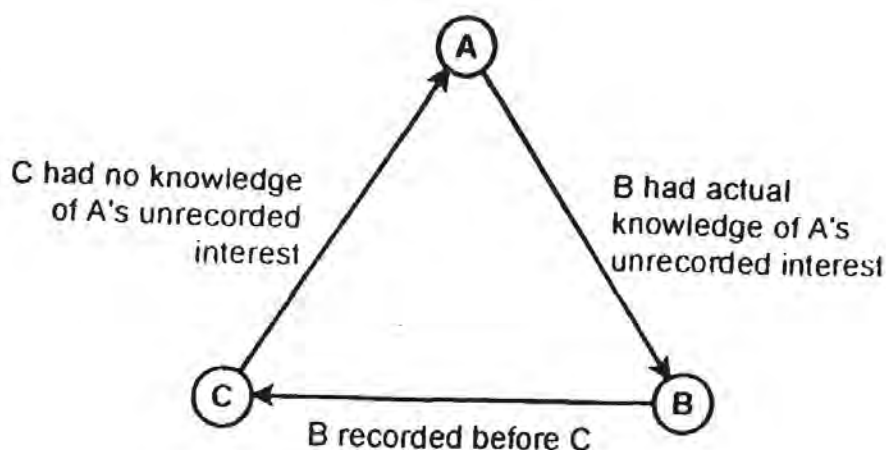


Figure 6

Articulating a rule to break this cycle seems more difficult than in the case of the subordination agreement, since no party assented to its creation. Arguably, however, A is "at fault" in the sense that she could have avoided this mess most easily (at least cost). The recording act focuses attention on this nonfeasance, even if it does not clearly make A liable. Thus, the majority of courts have applied the subordination rule against A in this case as well. B gets paid first, then C, and A is last in line.¹³⁵ The modern trend continues in this direction.¹³⁶

While no circular priority cases under Uniform Commercial Code recording systems¹³⁷ have yet been litigated,¹³⁸ scholars have found that the Code contains a number of rules that can give rise to such difficulties. This article briefly explains two examples, to give readers a flavor for circular priorities

135. See 2 GILMORE, *supra* note 120, § 39.4.

136. See *Pulawski Sav. & Loan Ass'n v. Aguiar*, 415 A.2d 365 (N.J. Super. Ct. Ch. Div. 1980) (rejecting another approach and adopting subordination rule); *In re 250 Bell Road*, 388 A.2d 297 (Pa. 1978) (switching from first in time rule to subordination rule).

137. All the examples considered *infra* involve, at least in part, security interests under Article 9 of the UCC.

138. Previous scholarship found no such cases. See Sutherland, *supra* note 117, at 519 n.15. Current research has similarly failed to unearth them. For a discussion of why no UCC circular priority cases have been litigated, and why eventually such cases are likely to surface, see *id.*

under the UCC.¹³⁹

First, circular priorities involving fixtures¹⁴⁰ can arise because the UCC permits filings in both the local real estate records and in the state "Article 9" records.¹⁴¹ While the former is effective against all subsequent interests, the latter gives no priority over subsequent real estate interests. Thus if, (i) A obtains a security interest in an electric range installed on Blackacre, but only files in the Article 9 records; (ii) B obtains a personal security interest and files in the local real estate records; and then (iii) C obtains a *real estate* interest over Blackacre (e.g., a mortgage); we get the now-familiar set of circular priorities:¹⁴²

139. For a more detailed explanation of these examples, along with other sets of UCC rules giving rise to circular priorities, see Stephen I. McIntoch, Note, *Priority Contests Under Article 9 of the Uniform Commercial Code: A Purposive Interpretation of a Statutory Puzzle*, 72 VA. L. REV. 1155 (1986); Sutherland, *supra* note 117, at 518-19; White, *supra* note 121, at 232-33; see also Sherman S. Hollander, *Imperfections in Perfection of Ohio Fixture Liens*, 14 W. RES. L. REV. 683, 691 (1963).

140. "The law of fixtures deals with relationships to objects that fall in a gray area between 'real property' and 'personal property.'" 5 POWELL & ROHAN, *supra* note 129, ¶ 648 (containing a collection of cases on disputed items). See also J. H. Crabb, Annotation, *Carpets, Linoleum, or the Like as Fixtures*, 55 A.L.R.2d 1044 (1957 & Supp. 1987); R. P. Davis, Annotation, *Electric Range as Fixture*, 57 A.L.R.2d 1103 (1950 & Supp. 1994); L.S. Tellier, Annotation, *Sprinkler System as Fixture*, 19 A.L.R.2d 1300 (1951 & Supp. 1996).

141. See U.C.C. § 9-313(1)(b) (1995).

142. For a different set of fixture rules leading to cycles, see Hollander, *supra* note 139, at 691 (arguing that the fixture lien provision of the 1974 version of U.C.C. § 9-313 gave rise to circular priorities).

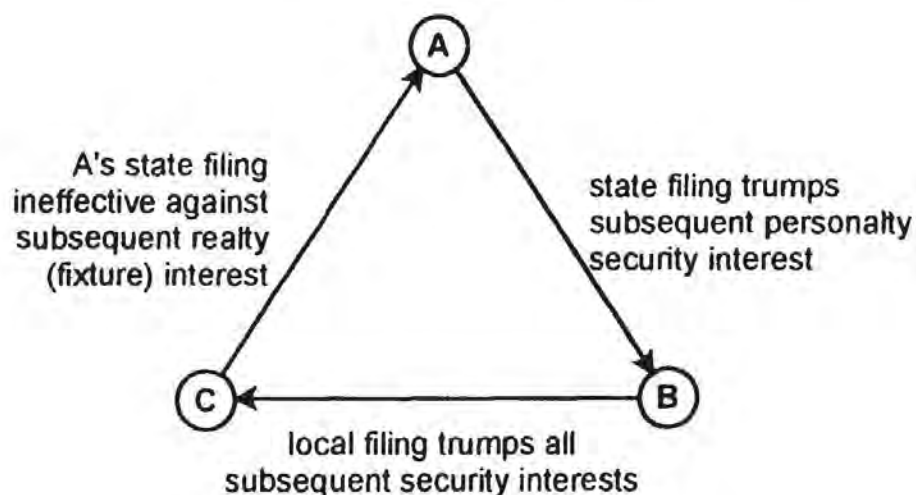


Figure 7

The second example involves the interaction of the consignment rules of UCC Article 2 with Article 9 security interests. When an owner (the "consignor") consigns goods for resale by a seller (the "consignee"), he must take steps to preserve his rights as against other creditors of the consignee.¹⁴³ In addition to filing to notify future creditors, the consignor must also give direct notice to existing inventory-secured creditors.¹⁴⁴ A cycle can then arise if consignor A gives notice to existing creditor B but not to existing creditor C, and B filed before C, then A will trump B, who in turn trumps C, who in turn trumps A.

c. Interaction of State and Federal Law

i. Federal Tax Liens

Even if state rules somehow prevent the possibility of circular priorities, their interaction with federal law may lead to the same conundrum. Historically, one of the most common instances of circular priorities arose in the interaction of federal

143. See U.C.C. § 2-326(3) (1995).

144. See U.C.C. § 9-114 (1995).

and state tax liens.¹⁴⁵ Before the Federal Tax Lien Act of 1966,¹⁴⁶ federal tax liens had priority over “inchoate” local property tax liens — essentially, those not reduced to judgment for a specific amount.¹⁴⁷ Congress, however, had subordinated federal tax liens to recorded security interests.¹⁴⁸ Since the usual state law rule is that property taxes trump even security interests, the following set of circular priorities often arose:



Figure 8

While the 1966 Act *inter alia* solved this particular problem by subordinating federal tax liens to property tax liens,¹⁴⁹ it by no means removed the possibility of circular preferences as a result of grafting federal tax liens onto state priority rules. Federal tax liens still enjoy priority over most “inchoate” liens, and as a result, courts still must deal with cases like *In re Stump*.¹⁵⁰ In *Stump*, plaintiffs wives had dower rights that, under Ohio law, gave them priority over a judgment lien holder. Their dower rights were inchoate (their existence and amount had not been determined), however, and so the federal tax lien took priority. Finally, federal tax law gave the choate

145. See 2 GILMORE, *supra* note 120, § 39.3, at 1032.

146. 26 U.S.C. §§ 6321-23 (1954) (amended 1966).

147. See 2 GILMORE, *supra* note 120, § 39.3.

148. See *id.*

149. See 26 U.S.C. § 6323(b)(6)(A).

150. 193 B.R. 261 (Bankr. N.D. Ohio 1995).

judgment lien priority over the federal tax lien.¹⁵¹ If we substitute the inchoate dower claim for the local property tax lien in the figure above, and the judgment lien for the mortgage, we have a structurally parallel cycle of priorities.

ii. Bankruptcy

In bankruptcy law, as in tax law, Congress has enacted legislation to untangle circular priorities. Under § 67 of the old Bankruptcy Act,¹⁵² which gave certain unsecured claims (such as administrative expenses in bankruptcy) limited special priority, the following cycle frequently occurred:¹⁵³

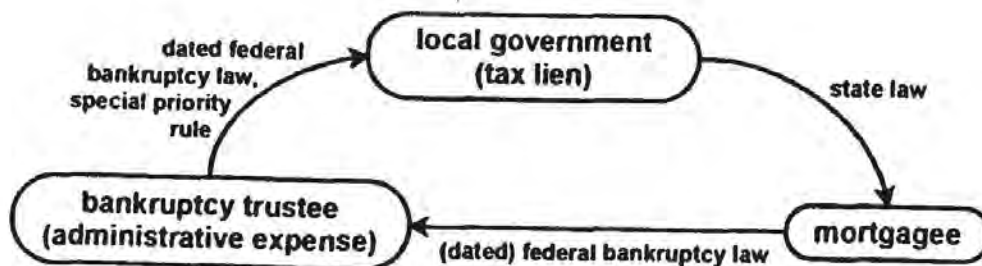


Figure 9

While Congress solved this problem in 1966,¹⁵⁴ it by no means rid the bankruptcy laws of the possibility of circular priorities. For instance, subordination agreements can combine with bankruptcy law to create cycles distinct from the "pure" subordination cycles previously discussed.¹⁵⁵ Suppose B ob-

151. See *id.* at 265.

152. 11 U.S.C. § 107(a)(3) (repealed 1977).

153. See 2 GILMORE, *supra* note 120, § 39.3.

154. While Congress amended § 67 of the old Bankruptcy Act in 1966, the same substantive result continues under the new Code, 11 U.S.C. § 724(b) (1993). See John C. McCoid, II, *Preservation of Avoided Transfers and Liens*, 77 VA. L. REV. 1091, 1110-11 (1991).

155. This example is loosely based on *In re Kors, Inc.*, 819 F.2d 19 (2d

tains a first mortgage on Blackacre, and A obtains a second mortgage.¹⁵⁶ While B makes the proper filing, A does not. Later, B subordinates his claim to A's for valuable consideration. If the owner of Blackacre then goes bankrupt, the trustee managing the estate can "avoid" A's mortgage since it was not filed ("perfected").¹⁵⁷ A maintains priority over B due to the subordination agreement.¹⁵⁸ Finally, to complete the circle, B has priority over the trustee since he properly filed his mortgage and put the world on notice of his interest.

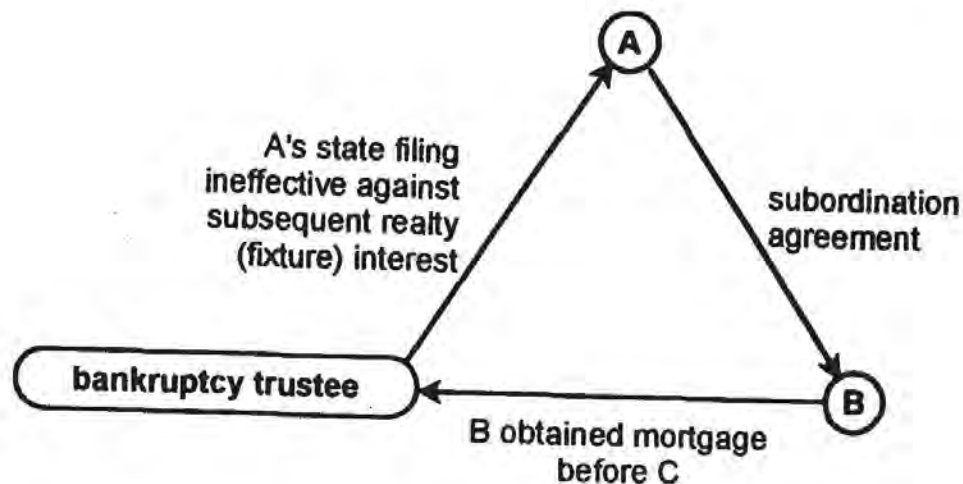


Figure 10

Cir. 1987).

156. GEORGE LEFCOE, *REAL ESTATE TRANSACTIONS* 531-34 (1993) (discussing the rights of junior lienors).

157. The trustee has this power under the so-called "strong-arm" provision of the Bankruptcy Code, 11 U.S.C. § 544(a)(1) (1993), which gives the trustee the power to avoid any and all interests over which a (hypothetical) judgment creditor would have priority on the day the bankruptcy petition is filed. Under 11 U.S.C. § 551 (1993), any interests avoided under § 544 are preserved for the benefit of the estate. *Accord Kors*, 819 F.2d at 23.

158. This is the central holding of *Kors*. See 819 F.2d at 23. While the trustee steps into A's shoes in terms of the security interest avoided, the trustee cannot step into A's shoes regarding side deals such as the subordination agreement.

Note that similar cycles can arise based on other provisions of the Bankruptcy Code. For instance, continuing the above example, even if A properly recorded his mortgage, the trustee would still have priority over A's interest if A received it "too close" to the bankruptcy filing; such "preferences" are voidable.¹⁵⁹ If B's mortgage was free of any such flaw, we have another case of circular priorities.

Indeed, the trustee's ability to avoid preferential transfers made on the eve of bankruptcy can produce cycles even without subordination agreements. It is possible that A will have a first mortgage that is voidable as a preference while B will have a second mortgage that is not so voidable.¹⁶⁰ If so, we immediately have the following cycle:¹⁶¹

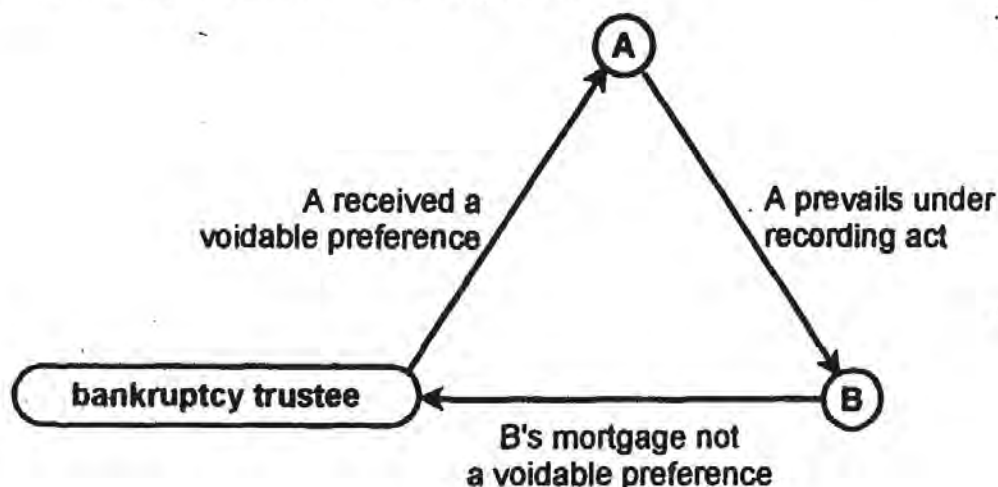


Figure 11

159. See 11 U.S.C. § 547 (1993).

160. This can happen in a variety of ways. For example, if B receives his second mortgage, not for an existing debt, but in exchange for new consideration, there is no voidable preference. See 11 U.S.C. § 547(c)(1) (1993).

161. This example is derived from McCoid, *supra* note 154, at 1091-92. McCoid argues that 11 U.S.C. § 551, which "preserves" voided transfers for the benefit of the estate, should be interpreted to break such cycles in favor of the trustee in cases where bankruptcy law only partially supersedes state law. See *id.* at 1093. It appears that most courts apply the subordination rule, discussed *supra* Part IV.A.1.a, in so breaking the cycle. See *id.* at 1091, 1093. McCoid discusses a host of other situations where circular priorities can arise under the Bankruptcy Code. See *id.* at 1104-10.

2. Exploring the Complexity of Cyclical Priorities

While the variety of situations in which circular priorities can arise may be a bit overwhelming at first, none of the examples discussed above appears very "complex." While courts have missed even such simple cases,¹⁶² it is nonetheless generally not difficult to detect circular priorities when the number of claimants is, for example, three.

The essential lesson of CCT, however, is that finding cycles becomes much more difficult when the number of parties increases by an even modest amount. This section illustrates the problem by iteratively constructing an example so complex that it is virtually impossible, even with the help of a computer, to find all of the circular priorities embedded in the fact pattern. Again, despite failure of earlier works to address this difficulty, finding circular priorities is obviously a necessary prerequisite to dealing with them.

Suppose that, in a notice jurisdiction, A obtains a first mortgage on Blackacre but fails to record. B and C obtain subsequent interests, with knowledge of A's mortgage and record. Then D and E obtain interests without knowledge of A's unrecorded lien. With only five parties we have a relatively complex picture with many cycles (how many?).

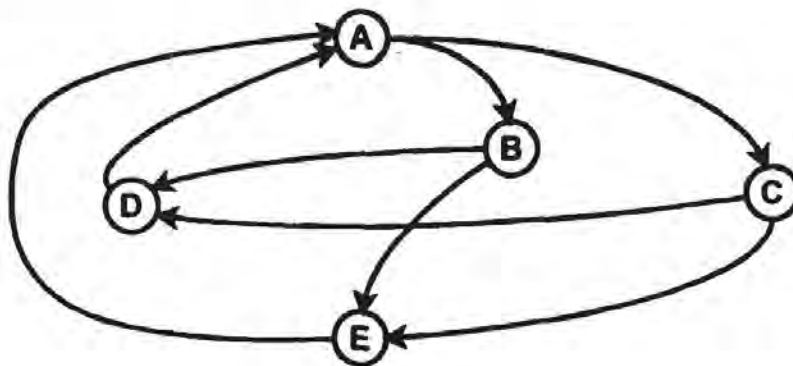


Figure 12

162. See, e.g., *Ursana v. Bank of Denver*, 354 P.2d 448 (Okla. 1960), cited in GILMORE, *supra* note 120, § 39.3, at 1032 n.1 (noting that the court failed to observe a simple cycle created by failure to perfect an appeal).

F then obtains an interest, fails to record, and bargains to subordinate C's interest to hers. Then G, with knowledge of F's interest, records. H, without knowledge of F's or A's unrecorded interests, obtains an interest and records. Now the situation is even more daunting:

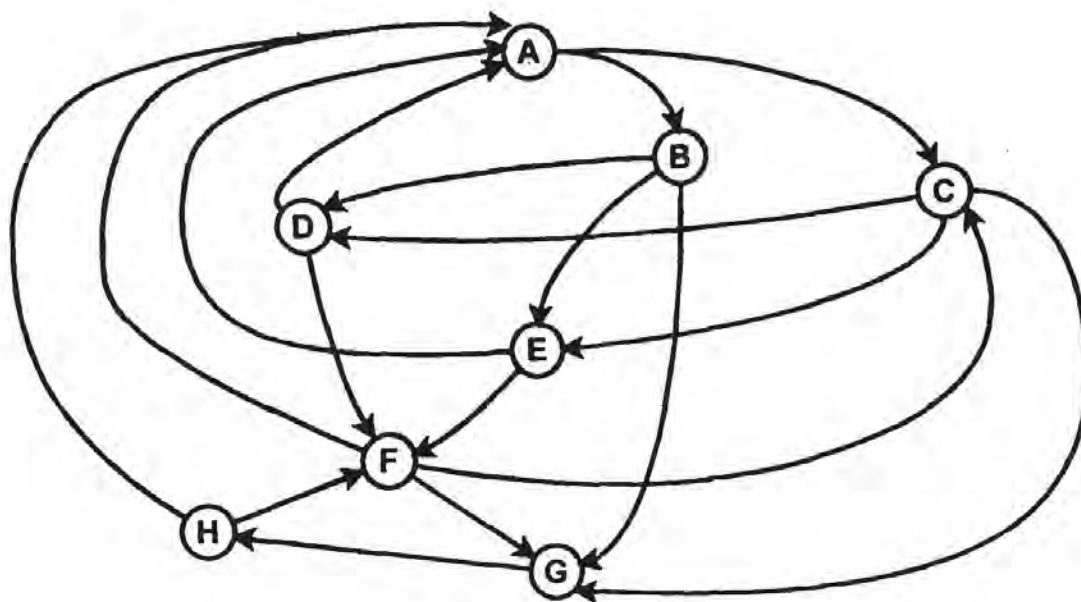


Figure 13

With only eight parties, and a modicum of dealings between them, it is now extremely taxing for a human to enumerate all the cycles, a seemingly necessary first step on the path to dealing with them. Moreover, if we add only a few more parties, we can construct an example that is so complex that a computer cannot list all the cycles in any reasonable amount of time:

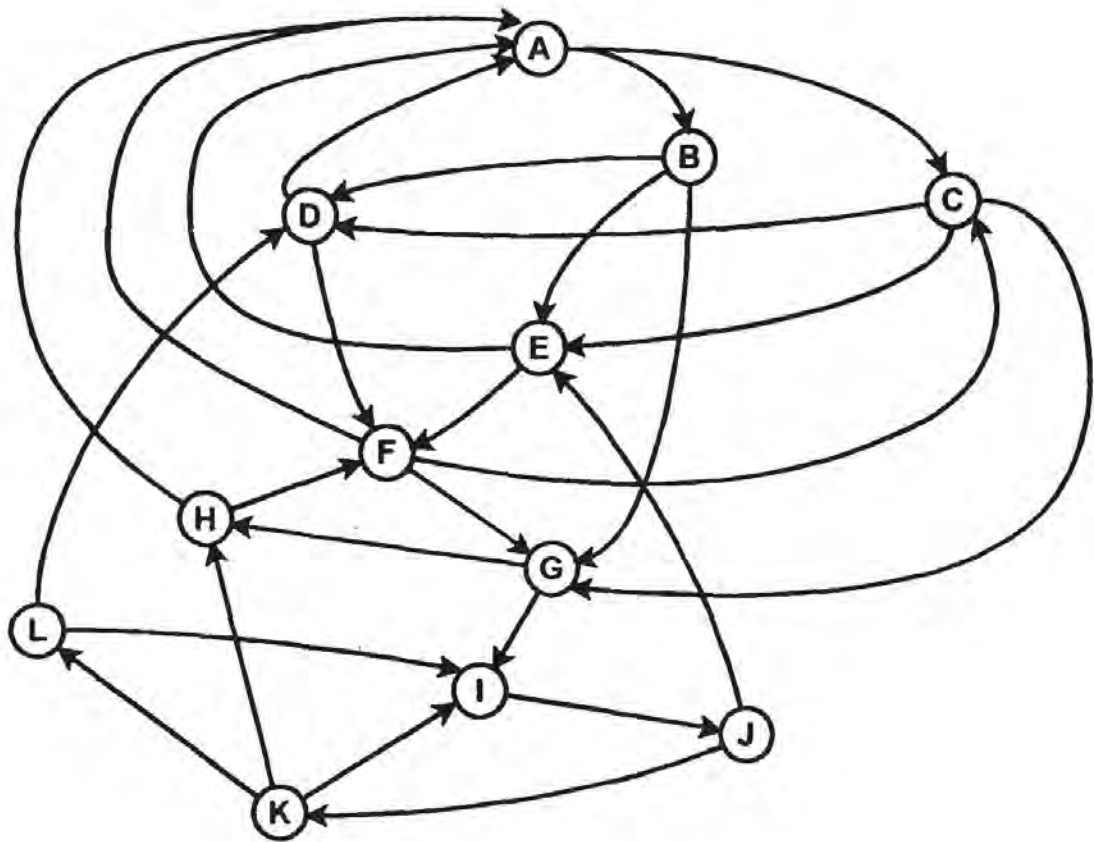


Figure 14

Things get even worse if different types of circular priorities can combine to generate complexity. This happens in practice. *In re Tuggle*,¹⁶³ for instance, involved a federal tax lien cycle on top of a state recording act cycle (that included state tax liens).¹⁶⁴

Before moving on to other legal rules that raise problems under CCT, the next two sections briefly present cycles that arise in different legal regimes.¹⁶⁵

3. Circular Corporate Voting Schemes

For obvious reasons, corporate law long forbade management from voting treasury shares (shares of its own stock owned by the corporation): "It is not to be tolerated that a Company should procure stock in any shape which its officers may wield to the purposes of an election; thus securing themselves against the possibility of removal"¹⁶⁶ If a corporation obtains a controlling block of its own shares, and if management could vote those shares, it would be impossible for the true owners of the corporation to oust management. Thus, all jurisdictions, in one way or another, forbid management from voting treasury shares and their voting equivalents.¹⁶⁷

163. 30 B.R. 718 (Bankr. N.D. Ga. 1983).

164. *See id.* at 719.

165. Not all problems involving circular relationships are intractable. A recent article, for example, demonstrated that it is possible to calculate the "true" owners of a corporation when some shares are owned by corporations in a cyclical relationship (a phenomenon discussed in the following section). *See* Stephen B. Land, *Strange Loops & Tangled Hierarchies*, 49 TAX L. REV. 53 (1993) (solving a cyclical ownership problem with a standard linear algebra technique that runs in cubic time).

166. *Ex parte* Holmes, 5 Cow. 426, 435 (N.Y. Sup. Ct. 1826).

167. *See* 5 WILLIAM MONROE FLETCHER ET AL., FLETCHER CYCLOPEDIA OF THE LAW OF PRIVATE CORPORATIONS § 2026 (perm. ed. rev. vol. 1990).

A corporation voting its own stock can be thought of as a one-element cycle.

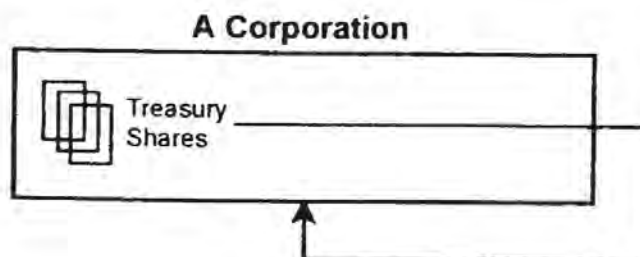


Figure 15

The next logical step, for management wishing to entrench itself, was to move on to a two-element cycle, by placing shares of the parent (call it A) in a wholly-owned subsidiary (B), thus evading the rule against voting treasury shares and yet retaining control over votes.

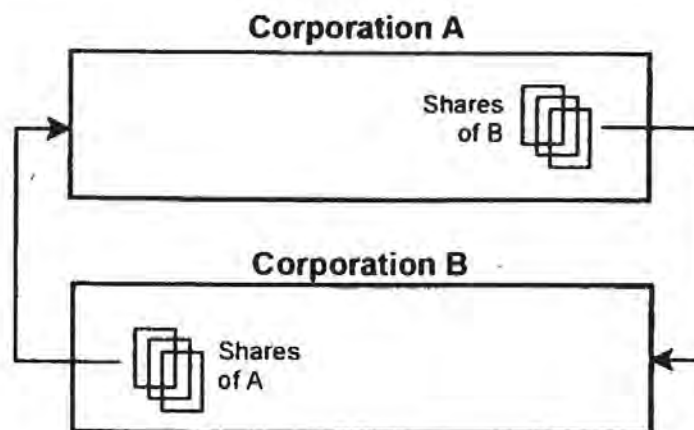


Figure 16

The courts had little trouble extending the rule against voting treasury shares to this slightly more complex example. "The actual substance of the affair is not changed by the fact that

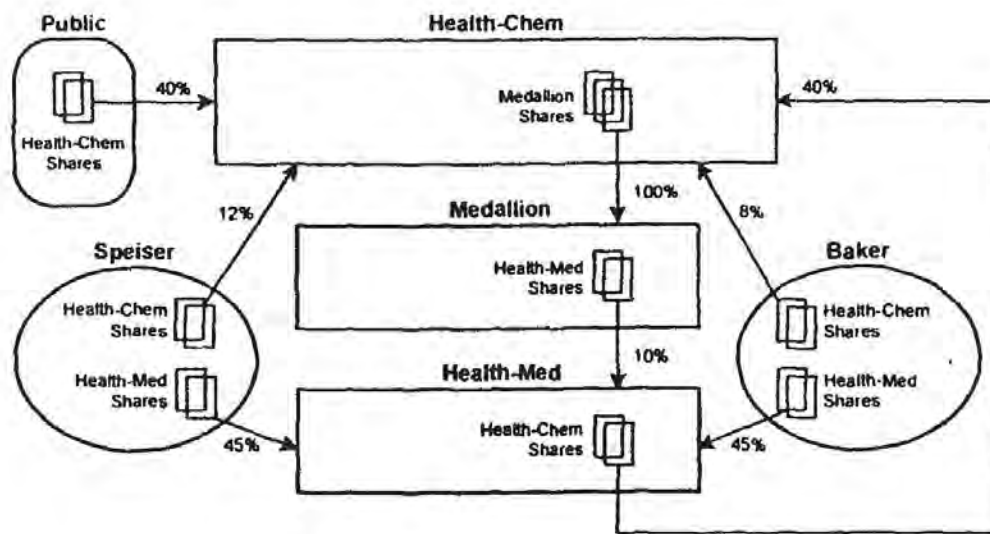
the nominal ownership of the shares in question remain [sic] in corporation B."¹⁶⁸ Judges extended this rule to cover both cases in which stock was owned by controlled (though not wholly-owned) subsidiaries,¹⁶⁹ and cases in which management salted away controlling blocks in a number of different entities.¹⁷⁰

168. *O'Connor v. International Silver Co.*, 59 A. 321, 322 (N.J. Ch. 1904), *aff'd*, 62 A. 408 (N.J. 1905).

169. *See Italo Petroleum Corp. v. Producers Oil Corp.*, 174 A. 276, 278 (Del. Ch. 1934) (ruling that statute on treasury share voting, with words "directly or indirectly" applied to shares owned by a 99% owned subsidiary: "What can 'indirectly' mean unless it be some such thing as having stock belonging to the corporation held in some third party's name and having that third party vote it?"). *But see Kalmanovitz v. G. Heileman Brewing Co.*, 595 F. Supp. 1385, 1398-99 (D. Del. 1984), *aff'd*, 769 F.2d 152 (3d Cir. 1985) (finding relationship between entity holding shares and target corporation not close enough to infer that management of target controlled voting of its own shares).

170. In *Norlin Corp. v. Rooney, Pace Inc.*, 744 F.2d 255 (2d Cir. 1984), Norlin's management responded to a takeover offer by placing Norlin stock in both a wholly-owned subsidiary and in an employee stock ownership program (ESOP) controlled by Norlin's board. The court, finding that management "amass[ed] voting control of close to a majority of a corporation's shares in their own hands by complex, convoluted and deliberative maneuvers," *id.* at 265, invalidated both moves.

If a two-element cycle of ownership cannot fool courts, why not try three? That apparently was the thinking of the litigants in *Speiser v. Baker*.¹⁷¹ Together, Speiser and Baker controlled only a third of Health-Chem's stock at the start of the game but, through a series of machinations, they gained control of the corporation by creating two shell corporations in the following arrangement:¹⁷²



(simplified version of court's diagram)

Figure 17

Thus, while Speiser and Baker controlled only 20% of Health-Chem's shares directly, by placing additional shares in Health-Med, which they controlled, they were able to vote, in total, 60% of Chem's shares.¹⁷³ This setup was sufficiently subtle

171. 525 A.2d 1001 (Del. Ch. 1987).

172. This is a simplified version of the court's diagram, see *id.* at 1004, abstracting away from complications created by preferred and convertible shares.

173. See *id.* at 1003-05.

that the majority shareholders did not realize that they had been effectively disenfranchised for years. The scam came to light only after Speiser and Baker had a falling-out.¹⁷⁴ The court extended the rule against voting treasury shares to reach this case of extended indirect voting of treasury shares.¹⁷⁵

Trying to detect circular voting highlights the need to find all cycles in a system. Consider the following example:

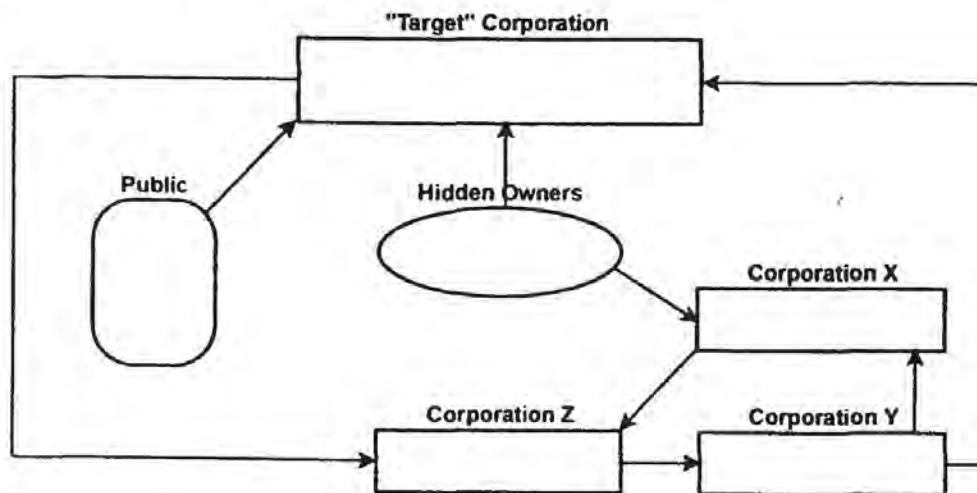


Figure 18

Control of "Target" Corporation may well depend on the ability of the hidden owners to control Y Corporation via a circular voting structure involving Corporations X and Z. Thus control of Target depends on a cycle that does not involve Target.

174. See *id.* at 1005.

175. See *id.* at 1012 ("[I]t is hard to imagine that a valid corporate purpose is served by perpetuating a structure that removes from the public shareholders the practical power to elect directors other than those supported by management.").

4. Criminal Conspiracies

The common law governing parties to a conspiracy provides a final example of rules that raise CCT complications in bigger cases. Determining the extent of a single conspiracy is a crucial battleground between prosecutors and defendants for a number of reasons. Prosecutors generally try to define a conspiracy as broadly as possible because, *inter alia*: each co-conspirator is liable for criminal acts in furtherance of the conspiracy by all other co-conspirators; statements by one conspirator during the course of the conspiracy are admissible against co-conspirators as a hearsay exception; and, if there is an overt act requirement, the act of one co-conspirator makes the rest subject to prosecution.¹⁷⁶

Conspiracies fall into two basic categories, along with a hybrid: chains, wheels, and chain-wheel conspiracies (which combine features of both).¹⁷⁷

Chain conspiracies typically involve a series of distributors of an illicit substance (narcotics or bootlegged alcohol). In order for the prosecutor to treat all the participants as members of a single conspiracy, she need not prove they all met and mutually agreed to a grand scheme. Rather, it is enough if each co-conspirator "by reason of [her] knowledge of the plan's general scope, if not its exact limits, sought a common [illegal] end."¹⁷⁸

Wheel conspiracies, as their name suggests, involve circular relationships, and this leads to CCT difficulties. At the center of the wheel is the *hub*, a defendant who had contact with many other members of the charged conspiracy. The hub's contacts with the other defendants form the spokes of the wheel.¹⁷⁹ To complete the analogy to a wheel, and thus permit treatment as a single conspiracy, the prosecutor must establish the rim of the wheel as well:

For a wheel conspiracy to be complete—i.e. for it to be fair

176. See JOSHUA DRESSLER, UNDERSTANDING CRIMINAL LAW § 29.07[B] (1987); see also WAYNE R. LAFAVE & AUSTIN W. SCOTT, JR., CRIMINAL LAW § 6.5(d) (2d ed. 1986).

177. See DRESSLER, *supra* note 176, at § 29.07[C].

178. *Blumenthal v. United States*, 332 U.S. 539, 559 (1947).

179. See DRESSLER, *supra* note 176, at § 29.07[C][1].

to say that there exists a single conspiracy that includes the hub and all of the spokes—there must be a rim around the wheel. That is, one must be able to draw a line around the wheel connecting the spokes. If this cannot be done then there is no single conspiracy; rather there exist [sic] as many [chain] conspiracies as there are spokes in which the membership of each conspiracy consists of the hub and the individual spoke.¹⁸⁰

In order to establish the rim, "circumstances must lead to an inference that some form of overall agreement exists,"¹⁸¹ there must be some "drawing of all [co-conspirators] together in a single, over-all, comprehensive plan."¹⁸²

As with circular priorities, it is not difficult to identify all the potential wheels (circular relationships) when the number of conspirators is small, or the structure of their relationships is simple. In a large international drug conspiracy, however, it is easy to imagine cases in which neither of these conditions holds. In addition to a large number of potential co-conspirators, it is possible that a wheel conspiracy will have multiple hubs, and in turn one of the hubs may be part of the rim in relation to the other hub or hubs. There may be multiple conspiracies, in which the hub of one conspiracy is a spoke in another. The courts might well find themselves facing diagrams no less complex than the daunting property recordation-cycle example previously presented.¹⁸³ In the criminal law, problems like this may be of constitutional dimension; a defendant in a complex conspiracy could plausibly argue that a rule too difficult to apply is void for vagueness.

B. Creditor Classes in Bankruptcy

CCT has implications for rules beyond those capable of generating circular relationships among the parties. The first such example involves classifying creditors in reorganizations under Chapter 11 of the Bankruptcy Code.¹⁸⁴ Any party proposing a

180. *Id.*

181. *United States v. Kenny*, 645 F.2d 1323, 1335 (9th Cir. 1981).

182. *Blumenthal*, 332 U.S. at 558.

183. *See supra* Part IV.A.2, fig. 14.

184. 11 U.S.C. § 1101 *et seq.* (1993 & Supp. 1996).

reorganization plan must divide creditors into classes whose members will receive equal treatment in most cases.¹⁸⁵ All members placed together in a class must have claims that are "substantially similar."¹⁸⁶

Classification is important since the voting and approval rules operate primarily in terms of the classes. To simplify a bit, each class must either: (i) be "unimpaired" — i.e., receive 100 cents on the dollar; (ii) vote to accept the plan, in most cases by two-thirds of the dollars of claims, and one-half of the number of claimants; or (iii) be treated fairly and not be discriminated against (in a so-called cram-down reorganization).¹⁸⁷

The following notation will help to conceptualize the classification process. First, we have the set of all claimants, {c1, c2, . . . }. Then we have a collection of all possible classes, {K1, K2, . . . }; the only restriction placed on these classes is that all members must be substantially similar.¹⁸⁸

K1 = {c1, c2}

K2 = {c1, c2, c3} (note: K1 and K2 are the only possible classes containing c1)

K3 = {c2, c3, c17}

·
·
·

The examples given here may seem counter-intuitive in two respects.

First, they reflect the fact that while claims must be similar before they can be classed together, case law has explicitly rejected the converse notion that all "substantially similar" claimants must be classed together.¹⁸⁹ Thus, the fact that the second possible class, K2, tells us that claims one, two, and three are similar, in no way rules out a plan's proposer from choosing a class that contains only claims one and two (i.e., K1).

185. See 11 U.S.C. § 1123(a) (1993 & Supp. 1996).

186. 11 U.S.C. § 1122(a) (1993 & Supp. 1996).

187. See 11 U.S.C. §§ 1126, 1129 (1993 & Supp. 1996).

188. See 11 U.S.C. § 1122(a) (1993 & Supp. 1996).

189. See *In re United States Truck Co.*, 800 F.2d 581, 585 (6th Cir. 1986).

Second, it may seem strange that:

- (i) K1 tells us that claim one is similar to claims two and three,
- (ii) K2 tells us that claims two and three are similar to claim seventeen, and yet
- (iii) K3, along with the note after K2, tells us that claim one is not similar to claim seventeen.

This merely illustrates that the relationship "substantially similar" among claimants is not transitive. An example should help illustrate why this is so. Consider defining two cities as being "close" if it takes less than twelve hours to drive between them in a car. New York and Pittsburgh are close, as are Pittsburgh and Chicago, yet Chicago and New York are not close. To the extent a plan classifies claims based on amount or other quantities akin to distance, the similarity relationship among creditors will not be transitive.¹⁹⁰

Taken together, the ability to place similar claims in separate classes and the nontransitivity of claims, give the drafter of a plan some flexibility in classification. A plan proposer's flexibility in classifying claims, however, is far from unlimited.

There must be some limit on the debtor's power to classify creditors The potential for abuse would be significant otherwise If the plan unfairly creates too many or too few classes, if the classifications are designed to manipulate class voting, or if the classification scheme violates basic priority rights, the plan cannot be confirmed.¹⁹¹

The most common "gerrymander" is to create an artificially large number of classes in order to insure that: (i) there is a class of creditors that, despite the fact that their claims are impaired, will vote for the plan; and (ii) any claims that are similar to this class, but whose votes against it would outweigh

190. See, e.g., 11 U.S.C. § 1122(b) (1993 & Supp. 1996) (permitting a separate class of small claims).

191. *In re Holywell Corp.*, 913 F.2d 873, 880 (11th Cir. 1990) (citations omitted); *In re Bryson Properties XVIII*, 961 F.2d 496, 502 (4th Cir. 1992) (adopting the language of *Holywell Corp.*); see also *In re Greystone III Joint Venture*, 948 F.2d 134, 139 (5th Cir. 1991) (stating that "thou shalt not classify similar claims differently in order to gerrymander an affirmative vote on a reorganization plan"), *modified*, 995 F.2d 1274, *cert. denied*, 506 U.S. 822 (1992).

the favorable votes, are placed in a separate class which is susceptible to cram-down or other unfavorable treatment.

Although it is not always clear what courts would label "gerrymandering," one seemingly simple way to eliminate the practice described in the previous paragraph would be to require the use of the smallest number of classes that includes every creditor.¹⁹² Unfortunately, this seemingly innocuous and straightforward-sounding "solution" is actually an intractable task.¹⁹³ Paradoxically, what seems like an easy solution is actually quite complex in larger bankruptcies.¹⁹⁴ As judges flesh out the meaning of "gerrymandering" in the context of classifying claimants in reorganizations, they will have to find a more workable definition of the term, at least for those cases with many creditors.

C. Impossibly Complex Terms in Contracts

In addition to the statutory schemes analyzed in the previous section, it is also possible for the parties to bring intractable contract disputes before a court. For instance, every year medical school graduates are matched with hospital residency training programs in a nationwide "marriage" process that attempts to optimize the pairings.¹⁹⁵

While simple versions are sometimes tractable, such "marriage" problems are quite often NP-c.¹⁹⁶ If so, whatever

192. If a creditor is included in more than one of the classes in this minimal set of classes, the plan's proposer has the power to choose the class in which to include the creditor, subject to the rule against gerrymandering. See *United States Truck*, 800 F.2d at 585-86.

193. This is known as the "minimum cover problem," and it was shown to be in complexity class NP-c. See Robert M. Karp, *Reducibility Among Combinatorial Problems*, in *COMPLEXITY OF COMPUTER COMPUTATIONS* 85 (R.E. Miller & J.W. Thatcher eds., 1972); see also GAREY & JOHNSON, *supra* note 98, at 222.

194. Bankruptcy is an area of the law where large cases are relatively common. See *infra* notes 207-10 and accompanying text.

195. See generally STANLEY ZASLAU, *MATCH SUCCESS: A STUDENT-TO-STUDENT STRATEGY GUIDE FOR APPLYING TO RESIDENCY TRAINING PROGRAMS IN THE UNITED STATES* (2d ed. 1994); AMERICAN MEDICAL STUDENTS ASS'N, *AMSA'S STUDENT GUIDE TO THE APPRAISAL & SELECTION OF HOUSE STAFF TRAINING PROGRAMS* (2d ed. 1986).

196. See HAREL, *supra* note 82, at 173; see also GAREY & JOHNSON,

matchings the residency programs propose are generally sub-optimal. If a medical student challenged the procedure as imperfect and cited documents aspiring to an ideal solution, a court would face an intractable problem in assessing the claim. It is generally not possible even to assess whether a proposed matching is relatively good, since the optimum is unknown. At a minimum, then, those conducting large-scale pairing operations should include a prominent disclaimer that they cannot generate an optimal solution, and further that they cannot even assess the relative merits of whatever heuristic technique they are using to produce a matching.

While complexity in such "marriage" problems, as in our previous examples, increases with the number of parties involved in the problem, trouble arises from a different source in large-scale construction (and other involved) contracts that include detailed agreements on scheduling successive tasks. "A major project may have thousands of activities, with each subcontractor generally performing a different activity."¹⁹⁷ Contractors face a daunting task in trying to calculate a feasible completion date with so many subtasks to coordinate; the task becomes more complex with each additional subtask.

Schedules in contracts have important legal implications. Perhaps the most prominent example is delay damages. Customers may sue the contractor, the contractor may sue subcontractors, and the subcontractors may in turn sue suppliers, for any and all costs that each faces as the result of another party's failure to adhere to the schedule.

The construction industry most commonly uses a scheduling technique called the critical path method ("CPM").¹⁹⁸

CPM . . . subdivides all the tasks into their most discrete components and differentiates between those construction activities that can't be started until other activities are finished and those that can be performed concurrently. Some delays in one activity might set off a chain reaction, ultimately preventing the project from being completed by the desired

supra note 98, at 221-23.

197. JUSTIN SWEET, LEGAL ASPECTS OF ARCHITECTURE, ENGINEERING AND THE CONSTRUCTION PROCESS § 26.05, at 560 (1994).

198. See MICHAEL T. CALLAHAN ET AL., CONSTRUCTION PROJECT SCHEDULING 27-28 (1992).

date. Any activity that would have such an effect is said to be "on the critical path."¹⁹⁹

The use of CPM is now widespread,²⁰⁰ and no wonder. Proper scheduling can reduce completion time by 40%, while poor scheduling can quadruple the time needed to put up a large structure.²⁰¹

Inevitably following widespread use in the real world, litigation over the application of CPM scheduling has become routine. "[I]t is rare to witness a claim litigation or arbitration case where planning and scheduling techniques, in particular CPM, are not part of the case."²⁰² Litigation over damages has been quite common: "[o]ne should not underestimate the relevancy of a formalized planning and schedule technique as it relates to claims—especially the quantification of damages aspect of claims."²⁰³ The courts have accepted CPM as a valid and enforceable contract term.²⁰⁴

Unfortunately, many CPM tasks relevant to scheduling large construction projects are intractable.²⁰⁵ As with matching

199. See *supra* note 156, at 898.

200. "The government now requires [CPM or similar] analyses on most, if not all, major construction projects." John M. Wickwire et al., *Critical Path Method Techniques in Contract Claims: Issues and Developments, 1974-1988*, 18 PUB. CONT. L.J. 338, 340 (1989). A survey taken in the late 1980s showed that 20% of all contractors used CPM or similar techniques for scheduling. See JAMES J. ADRIAN, *CONSTRUCTION CLAIMS, A QUANTITATIVE APPROACH* 192 (1988).

201. See CALLAHAN, *supra* note 198, at 1.

202. ADRIAN, *supra* note 200, at 225.

203. *Id.* at 192-93.

204. See, e.g., *Wilner v. United States*, 23 Cl. Ct. 241, 255-56 (1991) (relying heavily on CPM to determine fault for delay); *Pathman Constr. Co. v. Hi-Way Elec. Co.*, 382 N.E.2d 453, 460 (Ill. App. Ct. 1978) (citing the role of computers and techniques like CPM as aids in calculating damages); *Natkin & Co. v. George A. Fuller Co.*, 347 F. Supp. 17, 25 (W.D. Mo. 1972) (rejecting simpler approach unable to capture all the subtleties that CPM can model). But see *Titan Pac. Constr. Corp. v. United States*, 17 Cl. Ct. 630, 637 (1989), *aff'd*, 899 F.2d 1227 (Fed. Cir. 1990) (finding that CPM, though useful, is not always contractually binding).

205. See JEROME D. WIEST & FERDINAND K. LEVY, *A MANAGEMENT GUIDE TO PERT/CPM* 105-06 (2d ed. 1977) ("Scheduling projects with limited resources is a type of problem that mathematicians refer to as a

problems, the parties and any factfinder will have to rely on heuristic algorithms, and, again, there is no general way to assess the merits of a proposed solution. This raises a host of legal problems; perhaps most prominently, how a court should calculate delay damages since it cannot measure the length of the delay to the project due to a defendant's failure to perform on time. Courts rely on CPM to provide a benchmark.²⁰⁶ For some questions in sufficiently large cases, however, calculating this benchmark is simply not feasible.

D. Are These Really Problems? (Or, Why CCT Matters)

Cases large enough to raise CCT concerns may be rare. There are five reasons, however, why it is important for the legal community to recognize and understand the type of problem raised in this article. First, it is precisely in large cases that formal rules seem to offer the most guidance to judges and juries, but this article demonstrates that formal, mechanical rules do not always translate into useful ones.

Second, the number of large cases litigated is increasing, and the size of such cases seems to be growing. A recent bankruptcy case involved literally "hundreds of thousands of creditors,"²⁰⁷ and cases with hundreds or even thousands of creditors have become commonplace.²⁰⁸ Huge bankruptcies in industries that often have complex interrelationships among participants, like retail stores²⁰⁹ and finance,²¹⁰ increase the

large combinatorial problem . . . even [CPM], aided by the largest and fastest computers, can solve only small projects—those well under 100 activities."). *Id.* For a parallel result on the program evaluation and review technique (PERT), which allows for uncertainty in completion times, see GAREY & JOHNSON, *supra* note 98, at 218 (discussing "minimizing dummy activities in PERT networks").

206. "Schedules are an important part of proving (or refuting) delay claims because they provide a detailed medium for comparing and measuring time and intent." BARRY B. BRAMBLE & MICHAEL T. CALLAHAN, *CONSTRUCTION DELAY CLAIMS* § 9.1 (1986).

207. *In re General Dev. Corp.*, 84 F.3d 1364, 1374 (11th Cir. 1996).

208. See, e.g., Steve Kukolla, *Sycamore Executives Clean Up in Shutdown*, INDIANAPOLIS STAR, Jan. 18, 1996, at A1 (including "[m]ore than 1,000 creditors"); Jeffrey A. Trachtenberg, *Merksamer Unit of Hooker Files Under Chapter 11*, WALL ST. J., Oct. 26, 1989, at B12 (including "hundreds of unsecured creditors").

209. See *Campeau's Woes: Bankruptcy Petition Brings Fresh Risks for*

likelihood that creditor cycles will occur. As the frequency of such cases continues or increases, so does the likelihood that courts will find themselves trying to solve intractable problems.

Third, CCT raises the specter of parties using legal complexity to accomplish socially undesirable ends. Given the low expense of incorporation, for example, managers could construct a complex network of interrelated firms and hide a very long circular voting structure inside this corporate web. For example, shareholders never detected even the relatively simple three-entity cycle in *Speiser v. Baker*.²¹¹ There would be no systematic way for shareholders, or for a court, to detect, for instance, a fifty-entity voting cycle purposefully embedded in a hundred-entity network of interrelated corporations.²¹² To take another example, a bankruptcy court found that "[t]he difficulty in tracing the obligation of claims against [a number of creditors affiliated with the debtor] was . . . completely attributable to the labyrinth that [the debtor] created."²¹³ The law must be aware of just how impenetrable parties can make such a purposefully constructed labyrinth.

Fourth, drafters of legal rules should be cognizant that, simply because they sound precise, this in no way insures that sets of rules will be workable for large cases. Unless reasonably sure that large cases will never arise in the area that a rule covers, rule-makers should not create rules that will prove intractable in large cases.

Finally, the next section demonstrates that, in addition to exposing problems with specific laws and providing guidance for lawmakers, the CCT has broader implications for some recurring issues in legal complexity.

Allied, Federated, WALL ST. J., Jan. 16, 1990, at A1 (noting that "[t]he filing immediately affects . . . about 300,000 suppliers, hundreds of bondholders, and scores of other creditors.").

210. See Michael Siconolfi et al., *The End of Drexel*, WALL ST. J., Feb. 14, 1990, at A1.

211. 525 A.2d 1001, 1004 (Del. Ch. 1987).

212. See *supra* Part IV.A.3, fig. 18, for a smaller-scale example.

213. *Miami Ctr. Ltd. Partnership v. Bank of New York*, 838 F.2d 1547, 1552 (11th Cir. 1988) (citation and quotation marks omitted).

V. BROADER IMPLICATIONS OF COMPUTATIONAL COMPLEXITY THEORY

A. *The Number of Parties*

The Anglo-American legal tradition has always been uncomfortable with disputes involving multiple parties. It is still generally impossible to litigate a three-way conflict directly. The conflict must be formally broken down into a series of bipolar disputes²¹⁴ and class actions are a quite recent innovation.²¹⁵ CCT provides formal support for the intuitive belief that legal disputes will become unmanageable quite rapidly as the number of parties increases even modestly. Just as the number of subsets of a set increases exponentially as the size of the set increases, so too the number of possible opposing coalitions increases exponentially as the number of parties increases. Such multifaceted, multifactional disputes have traditionally been decided in the voting booth, not in the courtroom.²¹⁶ In crafting rules that invariably prune the number of parties, lawmakers and courts have seemingly recognized this problem.

214. Perhaps the biggest exception to this general rule is bankruptcy, which requires special rules and procedures precisely because it involves multiple claims on assets usually insufficient to satisfy those claims. Other exceptions include interpleader, FED. R. CIV. P. 22, and class actions. See *infra* text accompanying notes 216-17. For a view of many civil rights cases as three-cornered, see Douglas Laycock, *Due Process of Law in Trilateral Disputes*, 78 IOWA L. REV. 1011 (1993) (arguing that in some cases, due process demands that at least three distinct interests be involved in a single litigation).

215. See STEPHEN C. YEAZELL, *FROM MEDIEVAL GROUP LITIGATION TO THE MODERN CLASS ACTION* 10 (1987) (discussing the role of the class action in modern law).

216. Note that democratic political choice has its own set of formal problems. The possibility that, in attempting to aggregate citizens' preferences, voting may endlessly cycle between outcomes, none of which can garner a majority. See generally KENNETH J. ARROW, *SOCIAL CHOICE AND INDIVIDUAL VALUES* (1951). This "social choice" literature has grown rapidly in the legal community. For a skeptical assessment of many applications, see Maxwell J. Stearns, *The Misguided Renaissance of Social Choice*, 103 YALE L.J. 1219, 1257-86 (1994) (arguing that Arrow's criteria are an ideal, against which to measure, and then compare, collective decision-making bodies).

The motivating assumption behind class actions is that, in essence, there are only two parties to a dispute. Either side may have numerous individuals, but "the questions of law or fact common to the members of the class predominate over any questions affecting only individual members,"²¹⁷ and this strong symmetry between their interests permits the court to treat them as a great undifferentiated mass. Adding a new member to a class does not influence the complexity of the dispute, so courts can deal with arbitrarily large classes. If the courts treated each party as a new side, and had to compare each new party against all other parties, they would quickly face an intractably large number of disputes.

The rules governing class actions show in no uncertain terms that although the device is permitted, the legal system inexorably attempts to limit the number of "sides" in a case. The Manual devotes an extended section²¹⁸ to the management of class actions, and repeatedly focuses on cabining the number of disputants. First and foremost, it warns that "[r]arely should more than one [class] be certified . . ." ²¹⁹ A case with twenty classes would be at least as complex as a case with twenty individual parties. Even within a class, The Manual counsels limiting the number of perspectives a class may present. "In the interest of manageability, however, rarely should more than ten persons or firms be named as class representatives."²²⁰

Further, judges have the power to manage the number of counsel they deal with, e.g., by appointing lead counsel or committees of counsel.²²¹ By continually pruning the proliferation of potential sides in disputes, the courts prevent multi-sided litigation from becoming too complex. CCT provides a fresh and rigorous foundation for this behavior.

217. FED. R. CIV. P. 23(b)(3). This is not a requirement of all class actions. FED. R. CIV. P. 23(b)(1)-(b)(2) authorize class actions based on different grounds. That said, even these class actions must satisfy the less stringent requirement that "there are questions of law or fact common to the class." FED. R. CIV. P. 23(a)(2).

218. See THE MANUAL, *supra* note 44, § 30.

219. *Id.* § 30.15.

220. *Id.*

221. See Tidmarsh, *supra* note 45, at 1763.

B. Property Rights

Moving out of the context of litigation, CCT can provide additional support for economic rationality of private property. In a famous article, Harold Demsetz observed that one of the largest benefits of private property is internalizing costs. He focused on how the sharp rise in the value of beaver pelts, due to European demand, led to privatization of hunting grounds among the Montagne Amerindians of Canada's Labrador Peninsula.²²² Under the tribe's previous communitarian land practices, members had little or no incentive to hunt (or refrain from hunting) at a sustainable rate.²²³

There is no reason, in theory at least, why the Montagnes could not have agreed, as a community, to alter their hunting practices to maximize the long-term value of their beaver resources. In practice, of course, the rub is negotiating costs, a form of transactions costs. Changing the use of property in a purely communitarian system requires the consent of a majority, or perhaps of everyone. Under such a system, the number of potential coalitions and conflicts increases exponentially as the number of citizens rises, as does the complexity of figuring out schemes of compensating rearrangements.²²⁴ In contrast, private property negotiations generally involve only two parties, and thus pose fixed costs independent of the size of a society.

C. Fuller's Polycentric Tasks

In a famous article that anticipated both the problems with specific rules discussed earlier, as well as these broader concerns, Lon Fuller gave a penetrating justification for the courts' hesitance to take on *polycentric tasks* involving complex interrelationships between parties or things.²²⁵ His analysis

222. See Harold Demsetz, *Toward a Theory of Property Rights*, 57 AM. ECON. REV. PAPERS & PROC. 347 (1967).

223. See *id.*

224. Lon Fuller's concept of polycentric tasks, discussed *infra* Part V.C, in some ways anticipated this view of property systems. See *infra* notes 225-45 and accompanying text.

225. See Fuller, *supra* note 3, at 394-404. Fuller circulated this unfinished manuscript in the late 1950s, but it was not published until after

has striking parallels with CCT.

Defining adjudication as the right of litigants to engage in the "[p]resentation of proofs and reasoned arguments,"²²⁶ Fuller's basic question was, "[w]hat kinds of tasks are inherently unsuited to adjudication?"²²⁷ He used a number of hypotheticals to argue that polycentric tasks are beyond the competence of adjudication. His first example involved a will that left a large collection of diverse, valuable paintings to the Metropolitan Museum and the National Gallery "in equal shares."²²⁸ Fuller explained why effecting an equal division of the paintings is a difficult task:

[T]he disposition of any single painting has implications for the proper disposition of every other painting. If it gets the Renoir, the Gallery may be less eager for the Cezanne but all the more eager for the Bellows, etc. If the proper apportionment were set for argument, there would be no clear issue to which either side could direct its proofs and contentions.²²⁹

To see how closely this resembles the difficulties raised by CCT, consider the case of two paintings. Each gallery will take one, and there are no problems of "fit" among the set taken by each.²³⁰ If there are four paintings, there are only six possible divisions and it is not that difficult to examine each for artistic fit. If there are twenty paintings, however, there are almost 200,000 possible divisions into two sets of ten paintings each, and so finding the best one, or even a consistent one, by ex-

his death. He borrowed the term *polycentric tasks* from Michael Polyani. See MICHAEL POLYANI, *THE LOGIC OF LIBERTY: REFLECTIONS & REJOINERS* 171 (1951); see also Fuller, *supra* note 3, at 394. Fuller planned to expand the article into a book on "eunomics," meaning "the theory of good order and workable arrangements." *Id.* at 353. The phrase "workable arrangements" describes precisely the focus of CCT.

226. Fuller, *supra* note 3, at 363.

227. *Id.* at 393.

228. *Id.* at 394.

229. *Id.*

230. This discussion assumes the paintings are of roughly equal value. If they are not, the parties could agree on cash side-payments to equalize shares. This assumption seems warranted since Fuller is not focusing on the problem of dividing the collection of paintings into two groups of equal value. Rather, he is worried about the difficulty of dividing them into two sets that are artistically coherent.

haustive search is infeasible.

Indeed, Fuller seems to have stumbled intuitively across a task that is intractable.²³¹ He cites attempts to centrally determine prices and wages economy-wide,²³² or attempts to choose a football roster,²³³ as other instances of polycentric tasks. Generalizing the common element of these examples, Fuller argues that "the more interacting centers there are, the more the likelihood that one of them will be affected by a change in circumstances, and, if the situation is polycentric, this change will communicate itself after a complex pattern to other centers."²³⁴ Analogizing "interacting centers" to connected points forming a graph, Fuller's definition of polycentric sounds like a close analog to questions in graph theory that, as we have seen repeatedly, are replete with intractable problems.²³⁵

Although his article was written more than a decade before CCT developed, Fuller recognized that "[t]he relationship potentially affected by these decisions are *in formal mathematical terms* of great complexity—and in the practical solution of

231. We can think of each painting as a node on a graph, with edges connecting any two paintings that "fit" with each other. Trying to find larger and larger sets of paintings which *all fit with each other* is then the same as trying to find larger and larger sets of nodes that are all connected to each other. Such "totally connected" subsets of nodes are called *cliques* of graphs. Dividing up a graph into such cliques is known to be an NP-complete task. See GAREY & JOHNSON, *supra* note 98, at 193.

232. "Each . . . separate effect may have its own complex repercussions in the economy. In such a case it is simply impossible to afford each affected party a meaningful participation through proofs and arguments." Fuller, *supra* note 3, at 394-95.

233. "It is not merely a matter of eleven different men being possibly affected; each shift of any one player might have a different set of repercussions on the remaining players: putting Jones in as quarterback would have one set of carryover effects, putting him in as left end, another." *Id.* at 395. Note the similarity of this problem to attempting to solve the puzzle presented *supra* Part III.D. In both cases, it is easy to fit three or four players/pieces into place, but such a partial arrangement may be inconsistent with the best roster/puzzle solution.

234. Fuller, *supra* note 3, at 397.

235. See GAREY & JOHNSON, *supra* note 98, at 199.

them a good deal of 'intuition' is indispensable."²³⁶ In addition to recognizing the formal mathematical nature of polycentric (graph theory) problems, and their complexity, Fuller realized that formal, optimal solutions of such problems are generally unattainable. His "practical solution[s]" based on "intuition" are simply a definition of the *heuristic* solutions²³⁷ that computer programmers facing intractable problems must employ every day.

Fuller cited three reasons why adjudication is a poor forum to devise heuristic solutions to complex problems.²³⁸ First, the "[u]nexpected repercussions" of proposed remedies would undercut the effectiveness of remedies selected based on an ad hoc approach.²³⁹ Second, in trying to detect these hidden effects, Fuller worried that judges would commit procedural *faux pas*, such as 'sounding out' parties on possible remedies, perhaps at ex parte hearings, and making unsupported factual findings in order to support their judgments.²⁴⁰ Finally, Fuller worried that, having realized the intractability of a problem, judges might "reformulate the problem so as to make it amenable to solution through adjudicative procedures."²⁴¹ This yields the right answer to the wrong question.

Fuller's essay has been labeled as dated in light of the increasing willingness of the courts, especially the federal courts, to deal with disputes that look quite polycentric, such as desegregation cases, with far-reaching remedial tools such as the structural injunction.²⁴² While the normative debate over the use of the structural injunction is beyond the scope of this article,²⁴³ CCT strongly suggests that as a matter of positive

236. Fuller, *supra* note 3, at 398 (emphasis added).

237. See *supra* Part III.F.

238. See Fuller, *supra* note 3, at 401.

239. See *id.*

240. See *id.*

241. *Id.*

242. "Fuller's essay circulated for some twenty years but it was never modified to address the challenges posed—both positive and normative—by the litigative experience of the civil rights era. The essay was published in the late 1970s, but strikingly it always remained a statement of the late 1950s." ROBERT M. COVER & OWEN M. FISS, *THE STRUCTURE OF PROCEDURE* 508 (1979).

243. The seminal and still leading positive work on structural injunc-

theory, Fuller was ahead of his time, not behind it, in focusing on the difficulties that polycentric disputes pose for courts. Fuller and CCT frame the issue as follows: to the extent that the cases in which courts employ involved remedies, such as structural injunctions are bipolar, they are well-equipped to solve the problem; to the extent that the dispute is polycentric, however, the subtleties of framing a heuristic solution are probably beyond the judicial competence.

For example, Laycock²⁴⁴ characterizes *Martin v. Wilks*²⁴⁵ as a trilateral dispute. White firefighters (party one) sued their municipal employer (party two) for promoting minority firefighters (party three) based on race. If, as seems plausible, no other parties have a substantial interest in the dispute, then courts seem well able to fashion relief among three parties as a small departure from their run-of-the-mill bipolar cases. The number of "interacting centers" is small, the number of possible coalitions and side effects to consider is small, and thus trilateral disputes pose little difficulty from Fuller's or CCT's perspective.

As the number of classes with a substantial interest in a dispute increases even modestly, however, Fuller and CCT raise serious questions about the efficacy of adjudication. Consider, for example, school desegregation in a large metropolitan area. If the dispute really does break strictly along racial lines, it is bipolar and thus judicially manageable. Issues, however,

tions is OWEN M. FISS, *THE CIVIL RIGHTS INJUNCTION* (1978). Fiss continues to be a leading supporter of the remedial device. Owen M. Fiss, *The Allure of Individualism*, 78 IOWA L. REV. 965 (1993). See also Janet Koven Levit, *Rewriting Beginnings: the Lessons of Gautreaux*, 28 J. MARSHALL L. REV. 57 (1994) (examining use of the structural injunction using the *Gautreaux* litigation as a case study); Susan P. Sturm, *The Promise of Participation*, 78 IOWA L. REV. 981 (1993). For a critical appraisal, see Donald L. Horowitz, *The Judiciary: Umpire or Empire?*, 6 LAW & HUM. BEHAV. 129, 129-31 (1982) (criticizing the expansion in judicial participation in public policymaking which has come incrementally and on an ad hoc basis); Robert F. Nagel, *Separation of Powers and the Scope of Federal Equitable Remedies*, 30 STAN. L. REV. 661, 664 (1978) (arguing that the doctrine of separation of powers "impose[s] limitations on the authority of federal courts to undertake executive and legislative functions when ordering relief . . .").

244. Laycock, *supra* note 214.

245. 490 U.S. 755 (1989).

may be more complicated. Income, geographic location, and private school choice, for example, may be important attributes to determining the impact of a desegregation program on individual welfare. Middle-class African Americans in the suburbs may take a different view than poor inner-city African Americans. Poor suburbanites benefitting from stronger schools may be at odds with their city counterparts. Parents who send their children to private school, and pay for the public schools (and thus desegregation) but do not use them, may have interests largely unaffected by race or geography. These four factors (race, income, location, and choice of private education), yield a minimum of 24 separate classes that may have distinct interests. Moreover, there is a huge number of possible coalitions among these 24 groups.

Fuller in effect says that adjudication, like solutions to NP-c problems, does not scale well. As the size of a problem increases from two to twenty-two million, neither courts (humans) nor computers can keep up with the exploding level of complexity. While there is, at least according to grand economic theory, some set of policy choices that maximize social welfare (broadly defined), we would never trust a judge to issue an injunction setting forth every facet of social policy. The voting booth provides a procedure for sorting out the myriad of possibilities; voting is perhaps the ultimate heuristic solution to a formally intractable problem.

VI. JUDICIAL RESPONSES TO INTRACTABLE PROBLEMS

In dealing with these broader concerns regarding multiplicity of parties, property rights, and polycentric disputes, courts and commentators have been fairly sensitive to the difficulties posed by intractable problems. Returning to the more mundane topics previously discussed, such as circular priorities and classifying creditors,²⁴⁶ the question remains: what is a court to do when faced with a law and a fact pattern that produce an intractable problem?

The first reaction of Gilmore's judges, like bulls in rage, might be rather drastic. Simply eliminate such rules from common law and statute. This, however, is unnecessary. Credi-

246. See *supra* Part IV.

tor priority rules work in almost every case despite the possibility that a complex case will render them ineffective. Their benefits far outweigh the occasional cost of finding a different rule to deal with the most involved cases, and there is no need for a heuristic solution when an exact solution is attainable.

In the tough cases, however, the courts, like computer programmers, have no choice but to adopt a heuristic solution of one sort or another. Unfortunately, there is no single approach that makes sense in every complex case. The remainder of this section explores some of the issues relevant in choosing how to cut CCT's gordian knots.

In some cases, the court might return the problem to the parties. For instance, in creditor priority cases, the court could simply declare that it would entertain proofs from the parties and deal with whatever cycles they managed to expose. There are three problems with this approach. First, it does not make sense in other contexts, such as setting a rule to avoid gerrymandering of creditor classes in bankruptcy, where the court sets a rule and directs parties to perform a task that is effectively impossible in large cases. Second, it abdicates a court's responsibility to find facts. If the parties present all their interrelationships as a priority problem, for instance, they in some sense have presented all the facts. It is the court's duty, not the parties', to apply the law of priorities to those raw facts. Finally, letting the parties decide the issue provides relatively well-funded litigants with a decided advantage. The party with greater available resources can look longer and harder (i.e., hire more experts and employ more computer time) for favorable partial solutions (ignoring any unfavorable cycles that the other parties and the court might not find). Well-bankrolled litigants, of course, enjoy any number of advantages anyway (e.g., better counsel and experts), but courts should at least pause before compounding potential inequities.

If the intractable problem arises from a contract,²⁴⁷ the solution may involve no more than the application of standard contract theory. When parties insert an impossible condition in a contract, e.g., "set the price to be paid on Jan. 1, 1997 as the market price on Jan. 15, 1997," the courts, of course, cannot

247. See *supra* Part IV.D.

enforce the contract as written. They may either declare the entire contract a nullity (neither side having any enforceable rights), or try to substitute the true intent of the parties for the unworkable clause. In construction scheduling cases, for instance, a court could either appeal to a simpler, tractable scheduling mechanism designed to approximate its intractable cousin — much like a computer programmer's heuristic — or the court could simply dispense with formal approaches, apply a good-faith standard, and decide the dispute based on the intuition and sense of rough justice inherent in such a criterion.

It is important that a court be candid when it abandons a deterministic rule for a mushy standard like good faith in a complex case. When applying a common-law rule, this is no more than the duty of a court to declare the legal basis for its decision; e.g., "in this complex case we cannot apply our usual priority rules, and so, applying our accepted common-law power to create law, we adopt the following rule:" When the legislature ordains a rule that is intractable in large cases, the courts must engage in a bit of diplomatic statutory construction; e.g., "the legislature obviously could not have meant to impose an intractable task on us, and so we infer that in such complex case they intended us to" As in contract cases, the court should look to intent (here, that of the legislature) in trying to find a workable heuristic approximation to an unworkable law. Courts should not pretend they are doing the impossible. As another scholar has noted, attempts to hide complexity with "illusory scientific accuracy" and "illusory precision" tend to "obscure the relevant policy choices from public view."²⁴⁸ It is precisely such policy considerations that should drive the choice of a heuristic alternative to an intractable rule.

Unlike Fuller's polycentric disputes, judges do not seem incapable of weighing underlying policy issues and establishing rules that further them. Computer programmers searching for heuristic solutions to NP-c problems blend the skills of the artist with those of the scientist. Judges are expected to weigh policy goals and do rough justice every day, and dealing with

248. Flournoy, *supra* note 19, at 823.

intractable cases merely adds another entry to the list of cases in which they must exercise the art of judging.