The Conjunction Problem and the Logic of Jury Findings

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ABSTRACT

For several decades, evidence theorists have puzzled over the following paradox, known as the “conjunction paradox” or “conjunction problem.” Probability theory appears to tell us that the probability of a conjunctive claim is the product resulting from multiplying the probabilities of its separate conjuncts. In a three element negligence case (breach of duty, causation, damages), a plaintiff who proves each element to a 0.6 probability will have proven her overall claim to a very low probability of 0.216. Either the plaintiff wins the verdict based on this low probability (if the jury focuses on elements), or the plaintiff loses despite having met the condition of proving each element to the stated threshold. To solve this “conjunction problem,” evidence theorists have advanced such proposals as changing the rules of probability, barring probability theory entirely from analysis of adjudicative fact-finding, abandoning the procedural principle that the defendant need not present a narrative of innocence or...
nonliability, or dispensing with the requirement that the overall claim meet an established burden of proof.

This Article argues that the conjunction paradox in fact presents a theoretical problem of little if any consequence. Dropping the condition that proving each element is a sufficient, as opposed to merely a necessary, condition for proof of a claim makes the conjunction problem disappear. Nothing in logic or probability theory requires this “each element/sufficiency” condition, and the legal decision rules reflected in most jury instructions do not mandate it. Once this “each element/sufficiency” condition is removed, all that is left of the conjunction problem is a “probability gap,” an intuitive but ill-founded impression that the mathematical underpinnings of the conjunction problem are unfair to claimants. This probability gap is considerably narrowed by recognizing the probabilistic dependence of most facts internal to a given claim, and by applying the correct multiplication rule for probabilistically dependent events. Finally, this Article argues that solving the conjunction problem is an insufficient ground either to abandon probability theory as a useful analytical tool in the context of adjudicative fact-finding, or to reform decision rules for trial fact-finders.
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INTRODUCTION

An important theoretical inquiry that has long engaged evidence scholars involves the nature of legal fact-finding. Because the reconstruction of past events in litigation is inherently uncertain, the legal system adopts decision rules, known as burdens of proof, to permit fact-finders to reach decisions in the absence of complete certainty.\(^1\) The burdens of proof—“beyond a reasonable doubt” in criminal cases, “preponderance of the evidence” in most civil cases, and occasionally “clear and convincing evidence”—express probability thresholds that the law says must be reached to sustain a civil claim or criminal charge.\(^2\) Burdens of proof can thus be seen as a particular application of a broader question that philosophers would characterize as falling within the subfield of epistemology. This question is “under what conditions is belief justified?”\(^3\) In trials, we say in effect that belief in the defendant’s guilt or liability is justified when the burden of proof is met (or vice versa). It is axiomatic in our legal system that meeting the burden of proof requires evidence, but a more elusive theoretical question remains: *How do we know when* the burden of proof has been met? While as a practical matter judges and juries almost daily render seemingly acceptable decisions that the burden of proof in a case before them has or has not been met, evidence theorists continue to debate the theoretical underpinnings of this question.

A major point of contention in this evidence scholarship is what role, if any, probability theory actually plays, or should play, in telling us when the burden of proof has been met. One recent article claimed that “[m]ost evidence scholars believe that adjudicative fact-finding is fundamentally incompatible with mathematical probability,” and therefore the latter is not a usable guide to the burden of

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2. See id.
This skepticism is driven largely by the belief that “[a]pplication of mathematical probability in the courts of law engenders paradoxes and anomalies that are not easy to avoid or explain away.” Judging by the attention paid to it, the most serious of these paradoxes, posing the greatest challenge to probability theory, is “the conjunction problem.”

As framed by evidence scholars, the conjunction problem (also known as the “conjunction paradox”) goes like this: Suppose a plaintiff’s civil case consists of two elements—for example, defendant’s negligence and causation of plaintiff’s damages. The jury will be instructed that it must find each element to the degree of probability defining the burden of persuasion in civil cases: more than 50%, or 0.5 on a probability scale of 0 to 1. The jury may also be told—and if not, it will simply be a systemic background supposition—that a verdict for the plaintiff can be rendered only if the plaintiff’s overall claim meets or exceeds this same probability threshold.

So here is the problem: In basic probability theory, the “multiplication rule for conjunction” (also known as “the product rule”) holds that the probability of co-occurrence of two or more events is based on multiplying the probability of each. If occurrences $A$ and $B$ are mutually independent, then the probability of both $A$ and $B$ occurring together equals the product of the probabilities of $A$ and $B$ each

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4. See Allen & Stein, supra note 1, at 562.

5. Id. at 560.


occurring independently: \( Pr(A \& B) = Pr(A) \times Pr(B) \). Now suppose that the plaintiff proves each element to a 0.6 probability. The probability that both elements are true is \( 0.6 \times 0.6 = 0.36 \). If the jury focuses on the probabilities of “each element” separately without multiplying them, the plaintiff wins even though the overall probability of her claim is less than 0.5. If the jury instead focuses on the overall probability (0.36), the plaintiff loses even though she has met the burden of persuasion under the jury instruction to prove “each element” by a preponderance of the evidence (here, 0.6). Either way, there is an apparently irreconcilable tension between the two formulations of the burden of proof: one based on proving each element, and the other based on proving the whole claim.

The conjunction problem thus depends on the presence of two assumed conditions about how the law directs fact-finders to decide claims—specifically, the insistence both that the claimant wins by proving each element to the probability threshold (the “each element” condition), and that the claimant wins by proving her overall claim to the probability threshold (the “whole claim” condition). Notably, these legal decision rules are not themselves compelled by probability theory. But once they are both assumed, probability theory completes the conjunction paradox by imposing the axiom that the probabilities of the elements must be multiplied to determine the probability of the overall claim, which is their conjunction. The further axiom that those probabilities will always be between 0 and 1 dictates arithmetically that the product of the


9. The conjunction problem can also be formulated for criminal cases. Whatever numerical probability threshold is set for the beyond a reasonable doubt standard—say 0.9 for illustration—the product of a multi-element case with each element proved to 0.9 will multiply out to less than 0.9 for the whole case.


11. See infra note 35 and accompanying text.

12. See infra Part III.
probabilities will always be less than or equal to the lowest individual probability. Putting the “each element” and “whole claim” conditions together with the probability axioms, some claims will necessarily fall into a gap between these two conditions: that is, they will meet the “each element” condition but not the “whole claim” condition. We will call this the “probability gap.” Expositors of the conjunction problem seem to believe that a lot of claims will get lost in the probability gap. Hence, the conjunction paradox becomes the conjunction problem for conceptualizing the burden of proof as a predetermined probability threshold (such as 0.5 in civil cases and 0.9 in criminal cases).

The prevailing view seems to be that the conjunction problem cannot be solved as a formal matter, and that it instead needs to be worked around by a high-level theoretical solution. The most popular contemporary workarounds involve changing the rules of probability, banishing probability theory from the legal arena entirely, or abandoning the assumption that a claimant must meet a predetermined probabilistic threshold for his overall claim. Each of these theoretical fixes creates potential difficulties and costs that should make us want to be sure that the disease is worse than the cure.

In this Article, we argue that the conjunction paradox in fact presents a theoretical problem of little if any consequence. Dropping one aspect of the “each element” condition—the assumption that proving each element is a sufficient, as opposed to merely a necessary, condition for proof of a claim—makes the conjunction problem inconsequential. In a very brief symposium essay written over thirty years ago, Professor Dale Nance made this observation, and pointed out that nothing in logic or probability theory requires this assumption. He observed further that its foundation in the rules

14. See infra Part III.
15. See Ronald J. Allen & Sarah A. Jehl, Burdens of Persuasion in Civil Cases: Algorithms v. Explanations, 2003 Mich. St. L. Rev. 893, 894-95, 897, 929 (“We suspect nobody can explain the [conjunction paradox] formally.”); Allen & Stein, supra note 1, at 563 (“Evidence scholars, including us, have tried to resolve this paradox or somehow explain it away.”).
16. See infra Part II.
17. See infra Part II.D.
of adjudication is ambiguous at best. Unfortunately, Nance’s argument has received relatively little attention, and has not been developed at length.

This Article takes up where Nance left off. We show that most jurisdictions treat proof of the elements in their jury instructions as merely a necessary (not a sufficient) condition for sustaining a claim, although the practice of using special verdicts in civil cases introduces a measure of ambiguity about this.

With the removal of the condition that proving each element is sufficient for the plaintiff to win, all that is left of the conjunction problem is the above-described “probability gap.” In this Article, we also show that the probability gap (again, as a theoretical matter) has been exaggerated by expositors of the conjunction problem, who have mistakenly presented the problem as if all or most elements of claims are probabilistically independent. By recognizing the probabilistic dependence of most facts internal to a given claim and applying the correct multiplication rule for probabilistically dependent events, the probability gap may be considerably narrowed. In any event, the probability gap presents a problem of perception. The intuitive belief that too many cases fall into the probability gap is not a logical problem nor a reason to abandon probability theory as

19. See id. at 950-52.
20. Nance has repeated the argument on a couple of occasions, in a succinct length similar to his original formulation. See Dale A. Nance, The Burdens of Proof 74-78 (2016) (describing his analysis of the conjunction paradox as a “digression”); Dale A. Nance, Naturalized Epistemology and the Critique of Evidence Theory, 87 Va. L. Rev. 1551, 1572-74, 1592 (2001); see also Richard D. Friedman, Infinite Strands, Infinitesimally Thin: Storytelling, Bayesianism, Hearsay and Other Evidence, 14 Cardozo L. Rev. 79, 97-99 (1992) (suggesting that the conjunction problem disappears if elements are considered after the probability of the whole claim, and that considering elements separately is “a helpful shortcut in reaching a verdict against the claim”); David Hamer, The Civil Standard of Proof Uncertainty: Probability, Belief and Justice, 16 Sydney L. Rev. 506, 527 (1994) ("[A] consideration of the actual process of proof reveals the conjunction problem to be something of an illusion. In practice the mathematical standard will require the plaintiff to construct a story, with probability level of greater than 50 per cent, which includes each of the required elements.").

Ronald Allen and Sarah Jehl claim to have refuted Nance by purporting to show that most jury instructions unambiguously reproduce the conjunction paradox. See Allen & Jehl, supra note 15, at 898 n.17, 922-29. Nance has responded, correctly in our view, that Allen and Jehl misinterpreted most of the instructions they discussed. Nance, supra, at 76 & n.200. In Part IV we show that the majority of jurisdictions do not clearly reproduce the conjunction problem in their jury instructions, though they may do so in their special verdict forms.

21. See infra Part IV.C.
22. See infra Part III.B.
23. See infra Parts III.C-D.
a useful analytical tool in the context of adjudicative fact-finding. Moreover, as we show in Part IV, using the multiplication rule for dependent events helps to illustrate the actual function of the “each element” jury instructions that have so bedeviled commentators on the conjunction problem.

The theoretical inquiry into whether and to what extent probability theory usefully applies to our understanding of litigation fact-finding is an important one. But the conjunction problem is merely a distraction from the real merits of that conversation. Much evidence scholarship “has been driven by efforts to domesticate” the various “proof paradoxes” arising out probability theory.24 Because the conjunction problem is the most salient of these proof paradoxes, it has had an outsized influence in this debate, perhaps without contributing a great deal to its advancement.25 Whatever the strengths and limitations of using probability theory to explain trial decision-making, the perceived need to solve the conjunction problem should not be a factor in the debate.

To be sure, there may be practical problems arising out of contradictory probabilistic approaches to jury instructions. These, however, are empirical and policy questions. We conclude by arguing that evidence theorists should hesitate before proposing to reform the decision rules for juries in order to “solve” the conjunction problem.26 Until empirical research has demonstrated that juries are confused, or that they reach systematically “wrong” results, we see no basis to treat the conjunction paradox as a problem that needs to be solved through theory-driven legal reform.

24. Allen & Jehl, supra note 15, at 894-95; see infra Part II.
25. We do not claim to address other “proof paradoxes,” at least some of which may have more relevance to epistemological questions in evidence. For example, the “Blue Bus” paradox directly raises the question of whether a thinly evidenced probability is sufficient to justify belief. See Laurence H. Tribe, Trial by Mathematics: Precision and Ritual in the Legal Process, 84 HARV. L. REV. 1329, 1340-41 (1971). Ronald Allen and Michael Pardo have shown that the Blue Bus problem is an example of a deeper problem: that the reliability of purely statistical evidence is dependent on often-indeterminate questions about the proper reference class. See Ronald J. Allen & Michael S. Pardo, The Problematic Value of Mathematical Models of Evidence, 36 J. LEGAL STUD. 107, 109-10, 113-14, 137 (2007).
26. See infra Part IV.D.
I. WHY DOES THE CONJUNCTION PARADOX RAISE A PROBLEM?

The fact that jury trials continue to muddle along despite the seemingly unsolved conjunction paradox suggests that it may not create a problem of great practical import. We are aware of no empirical studies suggesting that fact-finders award verdicts to undeserving claimants whose claims multiply out to a product less than 0.5 or 0.9, but who meet the burden of proof for the separate elements.27 Cases of juries actually engaging in such a multiplication exercise, or being instructed to, are virtually unheard of.

Even if fact-finders applied the multiplication rule, it is unclear whether it would work systematically to the disadvantage of plaintiffs or defendants, because the conjunction problem can be framed to disadvantage either side. No clear consensus exists among evidence commentators whether a plaintiff who proves each of three elements to a 0.6 probability should win, because he has proven each element by a preponderance, or lose, because his claim as a whole has merely a 0.216 probability.28 And prosecutors should stand at the greatest disadvantage, since the product rule would pose the greatest difficulties for reaching the “beyond a reasonable doubt” threshold, which is typically quantified as high as 0.9 or 0.95.29 Yet we are aware of no serious empirical argument that prosecutors are unduly hampered in obtaining convictions or that the conjunction problem produces a slew of unwarranted criminal acquittals.30 One might think that the failure of the conjunction

27. Complicated jury instructions or special verdict forms may tend to work against plaintiffs by confusing juries with undue complexity. See Elizabeth G. Thornburg, The Power and the Process: Instructions and the Civil Jury, 66 FORDHAM L. REV. 1837, 1886 (1998) (“The complexity of the special verdict also works against the party with the burden of proof and the party seeking to change the status quo, usually the plaintiff.”). This is a different problem from application of the product rule.

28. Compare, e.g., Clermont, supra note 6, at 1108 (plaintiff should lose), and Spottswood, supra note 6, at 260 (same), with Charles Nesson, The Evidence or the Event? On Judicial Proof and the Acceptability of Verdicts, 98 HARV. L. REV. 1357, 1389-90 (1985) (plaintiff should win). See generally Allen & Jehl, supra note 15, at 933 n.152 (noting this ambiguity created by the conjunction paradox).

29. Cheng, supra note 10, at 1256 (citing Brown v. Bowen, 847 F.2d 342, 345-46 (7th Cir. 1988)).

30. About 85% of criminal trials end in guilty verdicts in the United States. Eric Rasmusen et al., Convictions Versus Conviction Rates: The Prosecutor’s Choice, 11 Am. L. &
problem to materialize in the form of real-world consequences would suggest to scholars that there is a conceptual flaw in the way the problem is constructed.

Perhaps a theoretical problem with no real-world consequences can be safely ignored. On the other hand, as one leading scholar puts it, “the conjunction paradox will unavoidably pose at least some practical difficulties”—perhaps encouraging undue judicial reliance on element-by-element special verdict forms that impede holistic jury decision-making.31 The paradox “therefore remains troubling, and theorists twist themselves into pretzels trying to explain it away.”32 Moreover, the mistaken belief in a real-world conjunction problem reflects some persistent misunderstandings about the nature of adjudicative facts, the role of essential elements in a cause of action, and the extent to which the law already embraces intuitive and natural reasoning processes without the need for mucking about by theorists. Given the huge impact that evidence scholars have long had on the development of evidence law,33 it is always worth the trouble to carefully examine whether an apparent problem is a real one, lest proposed theoretical fixes coalesce into ill-advised reforms.

What creates the conjunction paradox? It is based on a model of decision-making that makes the following assumptions regarding legal claims, all of which are broken down by the substantive law into two or more “elements”:

1. The elements of any legal claim are viewed by logic and probability theory as conjoint events.

ECON. REV. 47, 50 (2009). Some reviewers of drafts of this Article have argued that this high conviction rate at trial reflects a selection effect, in that prosecutors settle out their weaker cases and try their stronger ones, which would predict a higher win rate. This argument is almost certainly exaggerated and probably wrong. First, cases go to trial when settlement negotiations break down over probability and risk assessment divergences between the parties. Second, it is at least as likely that defendants take their stronger cases to trial. Third, if there were selection effects based on claimant case strength, those would operate in civil cases too, and would thus not explain the higher claimant win rates in criminal cases. In any event, the high conviction rate does not suggest a conjunction problem for prosecutors.

31. Clermont, supra note 6, at 1110.

32. Id.

2. The decision maker will assign each element of the claim a probability between zero and one based on the evidence in the case.

3. The overall probability of the claim is the product of the probabilities of the separate elements. This product will always be less than or equal to the lowest separate probability.

4. The “each element” condition: A necessary and sufficient condition for the claimant (plaintiff or prosecutor) to win the claim is that the probability of each separate element exceeds the probability threshold established by the applicable burden of proof.

5. The “whole claim” condition: If the probability of the overall claim is greater than the probability threshold established by the applicable burden of proof, the claimant should win the claim; if not, she should lose.

The source of the conjunction problem is easy to identify. Assumptions 1 through 3 apply axioms of probability theory to basic suppositions of the legal system, and need not be controversial. Assumption 5 also seems to be an uncontroversial assumption of the legal system. The conjunction problem arises from assumption 4, and more specifically, from the two italicized words, “and sufficient.” If those two words were omitted, assumption 4 would simply state an arithmetical corollary of assumptions 3 and 5.

In other words, the conjunction paradox arises out of the assumption that proving each element to a probability above the burden of proof threshold is a sufficient condition to find the defendant liable or guilty. Drop this assumption, and the problem goes away.\textsuperscript{34} To be sure, for the problem to go away, the separate probabilities of the conjuncts would have to exceed the threshold—by quite a lot where there are several elements—in order to multiply out to a product above the threshold. So there will be cases where the claimant loses

\textsuperscript{34} See Nance, \textit{supra} note 18, at 950 (arguing that the “each element” condition is necessary but not sufficient for a jury finding); \textit{see also} NANCE, \textit{supra} note 20, at 74-76 (noting how ambiguities in jury instructions can give rise to the conjunction paradox).
even though she proves each element to the threshold. But there will be no cases where the claimant wins without proving each element to the threshold. That is, the “each element” condition is necessary, but not sufficient, for the claimant to win.

It should be noted that the conjunction paradox is not a problem with probability theory, or even with the application of probability theory to legal reasoning. Rather, the problem arises from the apparent insistence on two decision rules that are conventions of the legal system: that a claimant wins if the probability of each conjunct element exceeds 0.5, but loses if the probability of the conjunction (her claim as a whole) is less than or equal to 0.5. Nothing in probability theory or logic compels this combination of decision rules. The sometime insistence of evidence theorists that the conjunction problem is a powerful indication that probability theory must be set aside as a description of legal reasoning therefore seems misplaced.35

For ease of reference, let’s break assumption 4, the “each element” condition, into two parts:

4a. Proving that the probability of each separate element exceeds the probability threshold established by the applicable burden of proof is a necessary condition for the claimant to win the claim.

4b. Proving that the probability of each separate element exceeds the probability threshold established by the applicable burden of proof is a sufficient condition for the claimant to win the claim.

To be clear, we are using the term “sufficient condition” in its philosophic logic sense: a sufficient condition for a state of affairs X is one that, if satisfied, guarantees that X is true.36 The conjunction

35. As Richard Friedman trenchantly put it, “I do not believe that any considered body of law precludes implementation of [the whole claim condition], and if there were the Bayesian approach would show that the law was wrong, not the other way around.” Richard D. Friedman, Answering the Bayesioskeptical Challenge, 1 INT’L J. EVIDENCE & PROOF 276, 280 (1997).

36. “X is a sufficient condition for Y’ means that if X is true, so is Y. That is, for Y to be true, it’s sufficient for X to be true.” ELLIOTT SOBER, CORE QUESTIONS IN PHILOSOPHY 387 (6th
paradox assumes that if the claimant proves that each separate element exceeds the probability threshold, her claim is necessarily “true” for legal purposes—she necessarily wins her case. That is, a jury finding that each separate element exceeds the probability threshold is equivalent to a finding in favor of the claimant. Thus, “sufficient condition” is not to be confused with the phrase “sufficient evidence” as used in the law of procedure and evidence, which means sufficient to permit a finding, but not to compel one. Confusingly, the legal term “sufficient evidence” supplies a logically necessary but not a sufficient condition for reaching a decision in favor of the party offering that evidence.

Many evidence theorists have demonstrated great resistance to dropping assumption 4b. As will be seen in the next section, they have offered various “solutions” or workarounds to retain it, including dropping assumptions 2, 3, and even 5. That is, they have tried to change or simply exclude the axioms of probability or, instead, to drop the requirement of an overall claim being probably true. What explains this resistance?

We think that two beliefs account for the resistance to dropping assumption 4b, and we believe that both are mistaken. The first is the belief that the law requires it. In Part IV, we will show that this is not the case. The second belief is a kind of intuition that dropping 4b is unfair to claimants. To win a simple negligence case with four elements—duty, breach, causation and damages—the plaintiff would have to prove each element to an average of more than a 0.84 probability (the fourth root of 0.5). This seems counterintuitive, because it feels as though the level of certainty that should be required for the essential elements should not be so high in the uncertain world of reconstructing litigated events. According to this intuition, such high probabilities of component facts will place unfair practical burdens on claimants; and in civil cases, those high

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37. See, e.g., 9B CHARLES A. WRIGHT & ARTHUR R. MILLER, FEDERAL PRACTICE AND PROCEDURE § 2524 (3d ed. 2017) (providing sufficient evidence to defeat summary judgment means showing that there is enough evidence on which a jury could reasonably find in favor of that party).

38. See infra Part II.
probabilities required of elements seem discordant with the more manageable “preponderance” standard. Let’s call this intuition a “probability gap” problem.

The probability gap is an appearance, rather than a logical issue, and a fallacious one at that. Axioms of logic and probability, such as the multiplication rule, are analytically true. The probability gap becomes a problem only if one chooses to contest the internal logic of probability rather than the intuitive perception of unfairness. If a claimant is expected to produce a specific, probably true narrative of his whole claim to show a defendant’s liability or guilt, there is no logical basis to resist the analytical axiom that the probability of the overall case is the product of the probabilities of its underlying facts. Perhaps our untutored intuition tells us that the probability of demonstrating negligence should instead be an average of the probabilities of the elements. But that is simply a math error.

We suspect that the probability gap, though purely perceptual and irrational, plays a significant role in giving the conjunction problem much of its emotional and intuitive traction. We may be wrong about that. And to be sure, expositors of the conjunction problem argue that there is a formal reason for believing that the probability gap is an analytical problem: that the law, and specifically jury instructions, require that proving each element to the stated probability threshold is a sufficient condition for the claimant’s victory. We will address that contention in Part IV. For now, suffice it to say that the perception of a probability gap should carry no weight in the analysis of whether a conjunction paradox exists and presents a problem.

39. Cf. A.P. Dawid, The Difficulty About Conjunction, 36 Statistician 91, 92 (1987) (attributing to conjunction problem expositor the “intuition” that “it is already very demanding on the plaintiff to require that every single one of the component issues be demonstrated ..., and that it would be unfair to demand the further establishment of the overall case”). For examples of the intuition, see, for example, L. Jonathan Cohen, The Probable and the Provable 58-61 (1977) (suggesting that, given the difficulty of proving each element to the probability threshold, the “whole claim” requirement is unfair to plaintiffs); and Clermont, supra note 6, at 1100 (“How did a strong case become a sure loser?”). Defenders of the use of probability theory to explain burdens of proof accept the mathematical fact that the individual probabilities of the conjuncts will have to be high. See, e.g., Friedman, supra note 35, at 282-83.

40. See, e.g., Allen & Jehl, supra note 15, at 922-29 (discussing jury instructions that require this conclusion).
The perception of a probability gap is magnified by a further, somewhat hidden assumption built into the conjunction paradox. Let’s call this one assumption 3’:

3’. The overall probability of the claim is the product of the unconditional probabilities of the separate elements. This product will always be less than the lowest separate probability.

This assumption that the probabilities are unconditional is based on the view—mistaken, as we will show—that elements of claims are probabilistically independent events. In fact, in most cases elements are probabilistically dependent, requiring the multiplication of conditional probabilities. Such conditional probabilities are still between 0 and 1, meaning that the product will still be less than or equal to the lowest probability. But the conditional probabilities are likely to be higher—often much higher—than unconditional ones, mitigating the appearance of an excessive probability gap between proof of the elements and proof of the claim.

II. PREVIOUS SOLUTIONS TO THE CONJUNCTION PROBLEM

Some, if not most, evidence scholars have come to doubt whether it is possible ever to “dispose[] of the conjunction paradox as a formal matter.” If “formal matter” in this comment means a solution based entirely on mathematics or on proof derived from the axioms of probability, we agree—and readers looking for that type of solution will no doubt be disappointed with our argument. Other evidence scholars addressing the conjunction problem have offered theoretical workarounds. These workarounds involve changing or dropping some combination of assumptions 2, 3, or 5 entailed by the conjunction paradox. In evaluating these workarounds, it is worth considering whether the conjunction problem is serious enough to justify the complexity of adopting alternative probability rules, barring the use of probability rules, or enabling claimants to win claims that may be far below a “more probable than not” threshold.

41. See infra Part III.B.
42. See infra Part III.C.
43. See Cheng, supra note 10, at 1256-57, 1257 n.5.
44. See Allen & Jehl, supra note 15, at 929.
A. Alternative Forms of Probability

Although L. Jonathan Cohen was not the first to notice the conjunction problem, he launched the ongoing debate surrounding it.45 Cohen argued that the conjunction paradox, along with other proof paradoxes, presented compelling evidence that standard probability theory—which he called “Pascalian”—is inapplicable to legal fact-finding, because (in his view) the former is concerned only with the frequency of recurring events and the latter with the reconstruction of unique past events.46 Cohen did not win a consensus among evidence scholars that his alternative “Baconian” probability theory, assuming he successfully axiomatized it, actually solved the proof paradoxes or offered a convincing account of adjudicative proof.47 Nevertheless, his general critique of standard probability theory has been quite influential among evidence scholars, many of whom have adopted his view that standard probability theory is inapplicable to adjudicative fact-finding.48

Kevin Clermont has advanced a different proposal for adopting an alternative probabilistic theory in his recent ambitious effort to solve the conjunction paradox. Clermont argues that fuzzy logic better fits the logic of jury decision-making and of the standard of proof than traditional probability theory.49 Fuzzy logic, according to Clermont, furnishes a quantitative judgment about how well a statement applies to a situation, rather than a probability of its applying exactly.50 For example, “Think of a somewhat sloppily drawn

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45. The earliest expositor of the conjunction paradox seems to have been Laurence Tribe in 1971. See Tribe, supra note 25. The debate heated up after Cohen’s 1977 book. See Spottswood, supra note 6, at 259 (citing Cohen, supra note 39, at 58-67).

46. See Cohen, supra note 39, at 58-67 (positing a holistic “Baconian” probability theory as an alternative to traditional “Pascalian” probability theory).


48. See, e.g., Allen & Jehl, supra note 15, at 929, 938 (arguing that probability of whole claim should not be viewed as the product of the probability of the conjunction of elements); Allen & Stein, supra note 1, at 560 (“[T]he relative plausibility approach outperforms mathematical probability operationally and normatively.”); Clermont, supra note 6, at 1080-86 (arguing that probability theory is better suited to predicting future events than to reconstructing past ones).

49. See Clermont, supra note 6, at 1085.

50. See id. at 1084-85.
circle: is it more appropriate to say (i) there is a 90% probability that it is a perfect circle or (ii) it has a .90 membership in the set of circles?51 Fuzzy logic seeks to take account of vagueness where the factual dispute turns not on our degree of belief in whether an event either did or did not occur but on the degree to which an occurrence fits a category.52 Says Clermont, “causation, consent, coercion, good faith, intent, and a host of other legal issues” occur by degrees rather than as all-or-nothing propositions.53 But at the same time, Clermont claims, “bivalent” probabilistic logic “is a special case of multivalent [fuzzy] logic,” meaning that fuzzy logic can accommodate traditional yes or no questions, such as whether a defendant was the perpetrator in a disputed identity case.54

According to Clermont, fuzzy logic solves the conjunction problem by replacing the multiplication rule with a “MIN” (that is, “minimum”) operator, which mandates that the probability of a conjunction is the lesser of the values of the conjuncts rather than their product.55 Let $F(S)$ represent the degree of certainty that fuzzy logic assigns to a statement $S$. The minimum rule says that $F(A \& B) = \min[F(A), F(B)]$. So a plaintiff who has shown his claim to have a 60% certainty of fault, and a 70% certainty of causation of damages, has proven his case to an overall certainty of 60%. Problem solved.

Clermont may well be right that fuzzy logic offers certain advantages for the analysis of vagueness, which is its traditional justification.56 We are less convinced by his attempted demonstration that fuzzy logic better captures the probability of a conjunction than standard probability theory, or that it entails abandonment of the multiplication rule for conjunctions. Clermont’s case for the MIN operator—the crux of his argument—seems to rest on the assumption that standard probability theory covers only a narrow range of probability statements.57 But, does the (traditional or “nonfuzzy”)

51. Id. at 1082.
52. See id. at 1080-81.
53. Id. at 1082.
54. See id. at 1084.
55. See id. at 1089-92.
57. See Clermont, supra note 6, at 1070 (noting standard probability theory’s difficulty with “partial truths”).
probability statement “I am 90% certain that this is a circle” mean exactly “I believe there is a 90% probability that this is a perfect circle”? Does it, correspondingly, exclude such meanings as, “this circle, though imperfect, seems to overlap with 90% of the characteristics of a perfect circle”? Clermont assumes that the answers to these questions are yes—requiring abandonment of traditional probability in favor of fuzzy logic—but we don’t see why that should be so.

Using this assumption as a starting point, Clermont tries to validate the MIN rule by offering various examples, of which the following is typical:

Imagine the law is trying to determine if Tom himself was at fault, that is, whether the perpetrator was Tom and whether the perpetrator was at fault....

....

...[W]e want to know if Tom’s fault is sufficiently true, not the chances of somehow discovering both the perpetrator certainly to be Tom and the perpetrator to be completely at fault.... If Tom is 60% likely the perpetrator, and the perpetrator is 70% at fault, the 60% figure means that it is 60% likely that Tom is surely the person who was 70% at fault. We thus have a 60% chance of Tom’s being legally at fault, just as the MIN rule would say.59

Clermont’s analytical move here is to round the 0.7 probability up to 1.0: by showing a 70% membership in the vague or fuzzy category of fault, which Clermont intuits is good enough for liability, the plaintiff “wins” that element and is entitled to a finding of fault.60 Clermont thus has reformulated the problem as the probability of liability given fault. If the two elements of the case are A (“Tom is the perpetrator”) and B (“the perpetrator is at fault”), and A = 0.6 and B = 1 (by Clermont’s assumption), then the probability of A and B under the product rule equals 0.6. Of course, the product rule generates the same result as the MIN operator whenever the probabilities other than the minimum equal 1. But this hardly

58. See id. at 1081-85 (discussing the advantages of fuzzy logic over traditional probability).
59. Id. at 1098-99.
60. See id.
proves the applicability of the MIN operator rather than the product rule.

Nor do we fully understand the basis for Clermont’s suggestion that the probability of an element should be rounded up to 1.0 whenever the plaintiff “wins” that element by proving it to a threshold greater than 0.5. In Clermont’s example, the fact that “Tom is the perpetrator” is a yes or no question, whereas “fault” is a category subject to vagueness or degrees of truth. These constructed characteristics may add a veneer of explanatory power to the example, but are really irrelevant, because Clermont asserts that the MIN operator applies to all efforts to reconstruct past events, regardless of the presence of vagueness. “[I]f you are looking back to the past, ... [y]ou are no longer trying to figure the odds of two things being sure, but rather how sure you are that one thing happened.”61 Indeed, all of Clermont’s examples seem to follow this pattern, treating elements other than the MIN as “somewhat sure” and “thus” equal to 1.62 That seems to assume a very unfuzzy world in which proven elements are 100% true and unproven ones 100% false.63 In any event, this assumption eliminates the need for fuzzy logic and the MIN operator: if all the elements with probabilities greater than 0.5 (or all but the lowest or minimum) are rounded up to 1, the conjunction problem disappears even under the multiplication rule. Indeed, it turns out that the same “rounding up to 1.0” move has been pressed by other writers who haven’t resorted to fuzzy logic.64

61. Id. at 1099.
62. See, e.g., id. at 1096-97 (arguing that “a .6 membership in the set of chairs and a .5 membership in the red set” produces a .5 membership in the set of “red chairs,” implying that the membership in the chair set is reflected in a multiplier of 1); id. at 1103 (assuming that in determining the probability of drawing two black balls, the color of the first ball is known, and therefore has a probability of 1); id. at 1103-04 (implying that fault proven to 70% should be converted to 1).

63. Apparently, in at least some versions of fuzzy analysis, the inputs are “fuzzified” and ultimately restated as “crisp” outputs. See Hajek, supra note 56. That does not seem to explain Clermont’s rounding up, however, which occurs prior to the calculation generating the output. See Clermont, supra note 6, at 1101 (demonstrating the MIN calculation).

64. For example, according to Charles Nesson, “Once jurors have decided that an element is probable, they are to consider the element established, repress any remaining doubts about it, and proceed to consider the next element.” Nesson, supra note 28, at 1386 (emphasis added). Repressing remaining doubts means rounding the probability up to 1.0. To the same
B. Elements, Not Claims

Formal efforts to solve the conjunction problem, like those above, have been much less common than pragmatic workarounds that accept the notion that the conjunction problem will never be solved. Several scholars have argued that we simply have to come to terms with the fact that in most cases, the conjuncts will multiply out to less than a 0.5 probability for the whole claim—even though the plaintiff should win many of those cases. One group of these scholars has argued that because the probabilities of the individual conjuncts are what really matter, we can simply ignore the probability of the conjunction taken as a whole.65

According to Charles Nesson, for example, the legal system simply “refus[es] to adopt the conjunction rule.”66 Instead it considers proof of the elements separately “to project its verdicts as statements about what happened,” in order to better achieve the deterrent aims of the substantive law.67 Nesson finds justification in an argument that prefigures the embrace of “inference to the best explanation” by evidence theorists.68

Application of the more-probable-than-not test to each element produces the most acceptable conclusion as to that element. The conjunction of these conclusions constitutes a story that is more probable than any other story about the same elements. Suppose, for example, that the elements of a story are A and B, and A (70%) is more probable than not-A (30%), and B (60%) is more probable than not-B (40%). The conjunction (A & B) (42%)

effect, see Alex Stein, An Essay on Uncertainty and Fact-Finding in Civil Litigation, with Special Reference to Contract Cases, 48 U. TORONTO L.J. 299, 311 n.27 (1998) (arguing that a favorable finding on an element converts to a “declaratory judgment,” presumably with a probability of 1.0). Stein has since disclaimed this view. See Allen & Stein, supra note 1, at 595 & nn.190-91.

65. For example, Saul Levmore has argued that permitting claimants to recover on less-than-probable civil claims is justified as compensation for the difficulty in persuading multiple jurors to agree on the same set of propositions, as reflected in the Condorcet Jury Theorem. See Saul Levmore, Conjunction and Aggregation, 99 Mich. L. Rev. 723, 734-45 (2001); see also Alex Stein, Of Two Wrongs That Make a Right: Two Paradoxes of the Evidence Law and Their Combined Economic Justification, 79 Tex. L. Rev. 1199 (2001) (arguing that the element approach may offset economic inefficiencies created by other evidence rules).


67. See id.

68. See infra Part II.C.
may not be more probable than its negation \((not-(A \& B))\) (58%). But the conjunction \((A \& B)\) (42%) is more probable than any other version: \((A \& (not-B))\) (28%), \(((not-A) \& B)\) (18%), or \(((not-A) \& (not-B))\) (12%). The application of the more-probable-than-not standard of proof on an element-by-element basis will produce the single most probable story.\(^69\)

Nesson’s example only works if we assume that the defendant must pick just one of the three “versions” of the “story” that establish “not-(A \& B).” But the “each element” instructions tell us that the defendant need only disprove one element to win.\(^70\) The defendant’s case can consist of any “story” that entails either \(not-A\) or \(not-B.\)\(^71\) The defendant thus need only prove the disjunction \((not-A)\) or \((not-B)\), and is thus entitled to add the probabilities of the three \((A \& B)\) scenarios.\(^72\) Moreover, even if the plaintiff produces more probable stories on individual elements, the defendant might produce “the single most probable story” by defending less than all the elements. Suppose the defendant produces a 40% probable story of \(not-A\), and produces no evidence on \(B\), but the jury credits the plaintiff’s case on \(B\) only to a 60% probability. In that case, the plaintiff will have proven each element but failed to “produce the single most probable story”: his liability story has a 36% probability, compared to the 40% defense story. To shore up his argument, Nesson would have to assume, contrary to prevailing legal rules, that a jury is required to give 100% credence to unopposed evidence offered by the claimant.\(^73\)

\(^69\) Nesson, supra note 28, at 1389-90 (footnote omitted).
\(^70\) See Allen & Jehl, supra note 15, at 897-902 (overviewing “each element” instructions).
\(^71\) See id. at 899 (noting that “each element” instructions require the plaintiff to prove every element to win a claim).
\(^72\) Where, as here, \(not-A\) and \(not-B\) are not mutually exclusive, \(Pr(not-A \text{ or } not-B) = Pr(not-A) + Pr(not-B) - Pr(not-A \& not-B).\) See Hacking, supra note 8, at 40-41. Using Nesson’s example, \(Pr(not-A \text{ or } not-B) = 0.3 + 0.4 - 0.12 = 0.58.\) Proponents of IBE have realized this problem and abandoned the implication that the defendant is required to pick just one of the possible stories negating \(A \& B.\) See Pardo & Allen, supra note 3, at 250-53 (IBE permits aggregation of disjunctive explanations); see also Pardo, supra note 6, at 1839 (same).
\(^73\) See 32A C.J.S. Evidence § 1641 (2017) (jury not required to believe uncontradicted evidence); see also Reeves v. Sanderson Plumbing Prods., Inc., 530 U.S. 133, 151 (2000) (suggesting that summary judgment cannot be based on even uncontradicted testimony from impeached or interested witnesses).
Even if we accepted Nesson’s “single most probable story claim,” the more obvious problem remains. As Clermont observes: “The implications are profound but boggling. Allowing recovery on a 36% showing of causation and fault [where each of those two elements is proven to a 60% level of certainty] is not only unfair but inefficient. How embarrassing for the law!” 74 Aside from its formal failings, Nesson’s argument also flies in the face of jury reasoning. A 36% chance that the plaintiff’s claim is true implies a 64% chance that it is untrue, according to probability theory. 75 The “story model” tells us that jurors seek out a single unifying narrative of liability, and it is hard to imagine them following instructions based on viewing each element in isolation and ignoring the probability of the whole—deciding in favor of a party whose story it disbelieves. 76 If Nesson’s response would be that the jury believes the plaintiff’s story more than any one of the defendant’s, his argument would merge with “inference to the best explanation,” which we address in the next Section. 77

C. The “Explanatory” or “Inference to the Best Explanation” Approach

The purpose of systems of philosophic logic and probability theory is to attempt to take statements about facts and beliefs made in natural language and to capture their meaning in formal structures that can be turned around to analyze the statements. The statement “X is probably true” can be formulated as an assertion that the probability that X is true exceeds the probability that X is false, or Pr(X) > Pr(not-X). This statement presupposes that X is more

74. Clermont, supra note 6, at 1108; see also Pardo & Allen, supra note 3, at 254 (focusing serially on each element produces a result that “does not distribute errors evenly among parties and therefore is unlikely to increase the accuracy of outcomes”).

75. See, e.g., HACKING, supra note 8, at 41.


77. Our categorization of the various approaches to solve the conjunction problem overlaps with, but differs from, Spottswood’s recent analysis, identifying “explanationist” and “novel mathematical” approaches. See Spottswood, supra note 6, at 293-94. We agree with Spottswood that Clermont’s and Nesson’s accounts can be reduced to the problematic claim that the probability of a conjunction is equal to its least probable element, which Spottswood calls the “Least Likely Element Rule.” Id. at 284.
probable than the sum of the probabilities incorporated in its negation, not-X.

Several evidence theorists have challenged this view by suggesting, in essence, that the statement “X is probably true” should be recast to mean that “X is the best explanation among the known alternatives,” irrespective of whether the probability of X can be said to exceed 0.5.\(^ {78}\) This theory is adapted from “inference to the best explanation” (IBE), an approach advocated by some contemporary philosophers,\(^ {79}\) and renamed the “explanatory” or “explanation-based” theory. Most closely associated with the work of Professors Michael Pardo and Ronald Allen,\(^ {80}\) Explanatory or IBE theory argues that litigation fact-finders decide cases by choosing the more plausible of the two competing explanations offered by the claimant and the defendant, instead of considering all possible explanations.\(^ {81}\) Rather than attempting to compare the strength of the claimant’s case (its narrative, or explanation of the evidence) to an abstract and elusive numerical standard of 0.5 probability (or 0.9 in criminal cases), the jury instead can use the purportedly more tangible criterion, the strength of the defendant’s case, as the benchmark.\(^ {82}\)

This strength is measured by non-numerical criteria: “coherence, consistency, coverage of the evidence, completeness, causal articulation, simplicity, and consilience (understood as the breadth of the explanation).”\(^ {83}\) Drawing on “story model” research, IBE proponents claim that their explanation-based theory offers the “best explanation” of trial fact-finding, arguing that it explains the actuality of jury decision-making better than probability theory or Bayesianism.\(^ {84}\)

\(^{78}\) See Pardo & Allen, supra note 3, at 223.

\(^{79}\) See generally Peter Lipton, Inference to the Best Explanation (1993); Gilbert H. Harman, The Inference to the Best Explanation, 74 Phil. Rev. 88 (1965).

\(^{80}\) See Pardo & Allen, supra note 3. Allen began adapting IBE to evidence law as early as the 1980s and has since then refined his theory. See, e.g., Ronald J. Allen, A Reconceptualization of Civil Trials, 66 B.U. L. Rev. 401, 403 (1986); Allen, supra note 76, at 381-82 (arguing that trials work best if the fact-finder is understood and required to choose the more believable of the conflicting narratives offered by the parties).\(^ {81}\) See Pardo, supra note 6, at 1839.

\(^{82}\) See Allen & Stein, supra note 1, at 568.

\(^{83}\) Id.

\(^{84}\) See id. at 571; see also Pardo & Allen, supra note 3, at 256, 258-60; Michael S. Pardo, The Nature and Purpose of Evidence Theory, 66 Vand. L. Rev. 547, 608-09 (2013).
There is certainly some force to the Explanatory theory argument that jurors decide cases holistically, that they do not assign precise numerical values to their assessments of probability, and that they fail to make rigorous probabilistic calculations, such as multiplying the probabilities of conjunct elements of claims.\textsuperscript{85} Whether the Explanatory account is or isn’t a better account of the burden of proof than the standard probability model is not a question we purport to answer here.\textsuperscript{86} Instead, we will analyze the claim that IBE solves, or avoids (or “tames”), the conjunction problem. IBE proponents claim that it does, and hold this out to be a significant selling point of their theory, making it superior to the standard probability account of the burden of proof.\textsuperscript{87}

\textsuperscript{85} See Pardo & Allen, \textit{supra} note 3, at 260.

\textsuperscript{86} It might be worth noting that IBE’s consistency with jury instructions is decidedly mixed. In our survey of jury instructions, including civil instructions from forty-seven jurisdictions (states and federal circuits) and criminal instructions from forty-five jurisdictions, we examined whether burden of proof instructions directed juries to assess the claimant’s case by a probabilistic threshold or by comparison to the defendant’s case. IBE, of course, supposes that juries do the latter. In civil cases, twenty-one jurisdictions defined the burden of proof by a probabilistic threshold: that plaintiff meets the burden by showing his claims to be “more probable than not” or the equivalent. Seventeen jurisdictions used a comparative standard, suggestive of IBE: either directing juries to decide in favor of the stronger case, plaintiff’s or defendant’s, or defining “preponderance of the evidence” as “the greater weight of the evidence.” Nine jurisdictions were ambiguous on this distinction. See \textit{infra} Part IV.C.

While IBE-type definitions of burden of proof represented a substantial minority of civil instructions (36%), they were almost nonexistent in criminal cases. See \textit{infra} Part IV.C. No jurisdiction defined “beyond a reasonable doubt” in terms suggestive of a best explanation comparing the strength of the plaintiff’s and defendant’s case, undoubtedly because the law does not require the defendant to present a case. See \textit{infra} Part IV.C. The closest thing to an IBE-type instruction in a criminal case was three jurisdictions (out of forty-five) directing juries to “compare and consider” the evidence. See, e.g., \textit{Judicial Council of California Criminal Jury Instructions }\S 221 (\textit{Judicial Council Advisory Comm. on Criminal Jury Instructions 2017} ) (“In deciding whether the People have proved an allegation beyond a reasonable doubt, you must impartially compare and consider all the evidence that was received during this trial.”).

\textsuperscript{87} See, e.g., Allen & Stein, \textit{supra} note 1, at 571 (“The relative plausibility structure of adjudicative fact-finding has three striking advantages. First, it solves the conjunction and all other paradoxes encountered by the frequentist account of juridical proof.”); Pardo, \textit{supra} note 84, at 608 (arguing that Explanatory theory provides normative guidance, in part by “resolving] any potential ‘conjunction’ problems”); see also Pardo & Allen, \textit{supra} note 3, at 253, 255 (arguing that while the conjunction problem “pose[s] a serious challenge to the [standard probability] approach,” the Explanatory account “does much better ... in taming the conjunction paradox”); Spottswood, \textit{supra} note 6, at 271 (reading Pardo and Allen to argue that avoidance of conjunction problem is a reason to adopt their theory over probabilistic account).
1. Avoiding the Conjunction Problem (1): The Holistic Solution

Explanatory theory proponents state their claim to avoid the conjunction problem with great assurance, but the details are actually unclear. According to Pardo and Allen’s leading account:

[An explanatory approach based on relative plausibility avoids the [conjunction] paradox. In civil cases, fact finders infer the best explanation of the evidence as a whole; in doing so they now have an accepted explanation that may or may not instantiate all of the formal elements of the claim. If it does, then the claimant ought to win; if not, not. In doing so, the formal paradox is effectively neutralized.88]

We refer to this as IBE’s “holistic” solution to the conjunction problem. Pardo and Allen’s statement seems to argue simply that jurors can and do fail or refuse to acknowledge and apply the multiplication rule to conjunctions (and so, therefore, can evidence scholars). Instead, jurors decide the claim holistically: they “first select an explanation” and only then do they “determine whether it includes the formal elements, rather than deciding the elements serially (which generates the paradoxes).”89

What does it mean for the explanation to “instantiate” or “include” “the formal elements”? Explanatory theorists don’t clearly explain, but presumably that requires looking at the elements individually to determine that the plaintiff has the best explanation on each element. For if, alternatively, an element had to cross a 0.5 probability threshold to be “included” or “instantiated,” then Explanatory theory would find itself in the odd position of saying that 0.5 is an unduly abstract standard for a whole claim, but a usable one for an element.

Since the best explanation must be evaluated at both the whole claim and the element level to determine “inclusion” or “instantiation” of each element, the fact-finder—contrary to Pardo’s

88. Pardo & Allen, supra note 3, at 255-56 (footnotes omitted). Since the defendant’s case does not have to include “all of the formal elements,” see id. at 256, a finding that the best explanation does not include one or more of the formal elements presumably means finding the defendant’s explanation to be the best available.
89. Pardo, supra note 84, at 608-09.
assertion—still must “decid[e] the elements serially.”90 The plaintiff must win at both levels. So unless the Explanatory theorists argue that the axioms of probability can be willed away,91 nothing in their account has done away with the conditions that work together to create the conjunction problem. First, we still have the potentially conflicting decision rules: that plaintiff wins by providing the best overall explanation, and that plaintiff wins by providing the best explanation as to each element. Second, we still have the axiomatic relationship between the probability of the elements and the probability of the overall claim, which is their product.

Disclaiming reliance on numbers does not make the conjunction problem go away, because better explanations are still those that both comprise elements and that are more probable overall. Notably, IBE proponents at times use numerical probability statements to illustrate their theory. Pardo, for instance, explains that if “a plaintiff offers a story that a reasonable jury concludes is 0.4 likely and the defendant offers a story that the jury concludes is 0.2 likely,” the plaintiff should win under IBE because “the plaintiff’s account is twice as likely to be true as the defendant’s alternative account.”92

If the plaintiff still has to win (“include” or “instantiate”) each element, in addition to winning the whole claim, the conjunction problem is replicated, unless providing the best explanation element-by-element is a necessary but not sufficient condition for a plaintiff victory. In other words, IBE’s “holistic solution” avoids the conjunction problem only to the extent that it replicates the account first articulated by Nance and elaborated upon by us below.93 That

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90. See id.

91. Despite some gestures in that direction by its proponents, see, e.g., Allen & Stein, supra note 1, at 602 (“[O]ur factfinding system refuses to guide itself by mathematical probability.”), we do not read IBE to make this claim. Thus, for example, Pardo and Allen argue that “a simpler explanation may be more likely [because] more complex explanations involve more auxiliary premises and background assumptions and, therefore, more places to go wrong.” Pardo & Allen, supra note 3, at 230 n.19. This is simply a plain language restatement of the multiplication rules for conjunction: the more conjuncts to be multiplied, the lower the probability of the conjunction. See Clermont, supra note 6, at 1096 (asserting that conjunction probability approaches zero as more elements are added).


93. See Nance, supra note 18, at 950; infra Part IV.
is to say, inclusion of the elements is a necessary but not sufficient condition for plaintiff’s victory, which also requires a holistic finding of probability.94 That the Explanatory theorists disclaim reliance on Nance’s argument suggests that they do not see matters this way,95 but that’s just a blind spot. All that Explanatory theory tells us is that fact-finders perform these two tasks—determining the most probable whole claim, and the most probable explanation of each element—without actually conducting a multiplication.96 But requiring a “holistic” finding that “instantiates” each element is the same as saying that there is a whole claim condition and an each element condition, and that the latter is necessary but not sufficient.97 The Explanatory theory thus seems to be groping toward Nance’s account. There is nothing wrong with Explanatory theorists embracing Nance’s account more consciously, but the bad news for them is that Nance’s account works even under the standard Pr(A) > 0.5 formulation of the burden of proof.98 In other words, nothing in the Explanatory theory’s “holistic” solution to the conjunction problem advantages Explanatory theory over the standard probability model.

2. Avoiding the Conjunction Problem (2): The Probabilistic Solution

Instead of what we label the “holistic” account, IBE/Explanatory theory may be seeking a different path to outdoing standard probability theory in its ability to avoid the conjunction problem. Explanatory theorists might argue that the Explanatory theory does not require the parties’ claims to occupy the entire probability space, equaling 1.0.99 Therefore, the plaintiff’s explanation A can be the best even when Pr(A) ≤ 0.5.100 Under the standard probabilistic conception of the preponderance of the evidence, the probability of

94. See Nance, supra note 18, at 950.
95. See Pardo & Allen, supra note 3, at 238-41 (considering their theorization incompatible with Nance’s work on the conjunction problem).
96. See id. at 255-56.
98. See id.
99. See Pardo & Allen, supra note 3, at 256 (criticizing standard probability theory for this requirement).
100. See id.
plaintiff’s claim $A$ must be “more probably than not true.” Standard probability theory interprets “more probably than not true” to imply a pair of propositions, in this case $A$ and not-$A$. By assumption, $A$ and not-$A$ are mutually exclusive, jointly exhaustive alternatives, which tells us that $Pr(A) + Pr(\text{not-}A) = 1$, by axiom. Therefore, $Pr(A) > Pr(\text{not-}A)$ if and only if $Pr(A) > 0.5$. Maybe the Explanatory account resolves the conjunction problem by modifying the “whole claim condition”—replacing the $Pr(A) > 0.5$ requirement with a rule that the probability of plaintiff’s case $A$ need only be greater than the probability of the defendant’s case $X$: $Pr(A) > Pr(X)$, where $Pr(A)$ can be equal to or less than 0.5.  

Note that, in this version, the Explanatory theory’s treatment of the burden of proof does no analytical work—that is, it fails to distinguish itself from the standard $Pr(A) > 0.5$ account—where the fact-finder considers the entire probability space. That space consists of the full range of scenarios summing to a probability of 1.0. In such a case, $Pr(A) > Pr(X)$ and $Pr(A) > 0.5$ are equivalent statements. To be sure, the range of scenarios supporting civil liability or criminal guilt, and their opposites, are theoretically infinite. But most of these scenarios in many cases will have a de minimis probability, so that the probabilities placed before the factfinder will sum very close to 1.0. In those instances, the standard $Pr(A) > 0.5$ account may well be a good approximation—the “best explanation,” if you will—of the burden of proof.

For IBE to have real theoretical bite, there must be a significant number of cases in which the probabilities placed before the factfinder add up to significantly less than 1.0. Pardo and Allen recognize this:

>T]he standard problem of trials is not to accumulate all the stories for the parties and see which collectively adds up to great [sic] than .5. Rather, the standard problem may be something more like the probability of the plaintiff’s case being .4, and the

101. See Hacking, supra note 8, at 40-41.
102. See id.
103. See Pardo & Allen, supra note 3, at 256.
104. See Friedman, supra note 20, at 92.
105. See id.
respective probabilities of the two [disjunctive] defenses each being \( .1 \). Viewed this way, IBE seems to avoid the conjunction problem by permitting jury verdicts in favor of the plaintiff when the product of the probabilities of elements is less than 0.5. Cases falling into the “probability gap” can still result in plaintiff victories so long as \( Pr(A) > Pr(X) \).

But this “probabilistic” version of Explanatory theory’s conjunction problem solution, like the previous “holistic” version, fails to improve upon the standard \( Pr(A) > 0.5 \) account. For the plaintiff to win with \( Pr(A) \cdot Pr(X) \) where \( Pr(A) \leq 0.5 \), one or both of two things has to be true. Either there must be a substantial probability attached to “unpresented” explanations—that is to say, explanations that have not been presented by the parties—or the defendant must be precluded from presenting a disjunctive case. Both of these consequences of IBE may undermine goals of the adjudication system and thereby impose unwanted costs.

Let’s first consider the problem of unpresented explanations. If plaintiff’s case consists of two elements each proven to 60%, the probability of plaintiff’s case is 0.36. To be a better explanation than defendant’s case, the latter must have a probability less than or equal to 0.35. With the explanations presented by the parties adding up to 71% probability or less, there must be nearly a 30% or greater probability “unaccounted for” at trial. That is, there must be unpresented explanations whose probability sums to nearly 0.3, or more. To change the scenario slightly, assume that plaintiff’s case consists of three elements each proven to 60%, so that the probability of plaintiff’s case is 0.216; for that to be the “best explanation,” defendant’s case must be less than or equal to 0.215, leaving a sum total of nearly 60% probability of unpresented explanations. If the bulk of these unpresented explanations favor the defendant, the risk of erroneous decisions seems rather high.

To be sure, IBE proponents sometimes say that the defendant should not benefit from unpresented explanations. In that event, IBE must either make a normative claim that fairness concerns

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106. Pardo & Allen, supra note 3, at 256.
107. See id.
counterbalance this risk of error, or make an empirical claim that the distribution of unpresented explanations cannot be known to favor either plaintiff or defendant, and should thus be disregarded. But if we rule out unpresented explanations, then IBE once again converges with the standard \( Pr(A) > 0.5 \) account, because it will have recreated, by fiat, a world for trial purposes in which \( Pr(A) \) and \( Pr(X) \) occupy the entire probability space that the jury may consider: \( Pr(A) + Pr(X) = 1 \).

Alternatively, IBE might bar defendants from disjunctive explanations. We have already seen this problem in Nesson’s account. If plaintiff’s case consists of two elements each proven to 60%, the probability of plaintiff’s case is 0.36. But defendant’s case as to each element has a probability of 0.4. If defendant’s case as to each element is permitted to be considered as a disjunction, the probability of defendant’s case overall is 0.64. Plaintiff loses this case under IBE’s decision rule \( Pr(A) > Pr(X) \) if we consider the whole claim, yet plaintiff has offered the best explanation as to each element. Viewed this way, IBE replicates the conjunction paradox.

The “relative plausibility” theory previously advanced by Allen has this problem, along with certain limitations both as a description of actual jury behavior and as a description of legal and evidentiary procedures. The fact that jurors need to be cautioned in every trial not to speculate—that is, to avoid filling in evidentiary gaps with their own hypotheses—suggests that it is quite natural

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108. See, e.g., Allen & Jehl, supra note 15, at 932-33 (suggesting conventional probability account impossibly requires plaintiff “to specify, and disprove, all the ways in which someone other than the defendant might have caused the result”); Pardo, supra note 84, at 593 n.197 (“To truly prove a case beyond 0.5 requires a plaintiff to disprove all the other ways the world could have been from what the plaintiff alleges,” which “is an impossible task.”).

109. Allen, who claims to be advancing IBE purely descriptively, would presumably incline toward the latter view, while Pardo argues for the normative desirability of IBE. Compare Pardo & Allen, supra note 3, at 226 (“[T]he primary focus of our explanation-based account will be descriptive and explanatory.”), with Pardo, supra note 84, at 598 (explanatory theory “provides normative guidance”).

110. See supra notes 69-77 and accompanying text.

111. This is the sum of the two probabilities minus their product, 0.8 - 0.16 = 0.64. See supra note 72.

112. See Allen, supra note 80, at 425-31; Allen & Jehl, supra note 15, at 936-43.

for jurors to want to go outside the parties’ presented narratives when deliberating. Indeed, the reason why trial practice manuals (and the “story model” research) so strongly advise trial practitioners to tell comprehensive stories is precisely because jurors will naturally tend to make up facts that are not provided to them, if doing so strikes them as important to making the story conform to the qualities of good explanations, mentioned above.114

Our expectation that jurors evaluate evidence by comparing it to generalizations based on their common sense and experience further suggests that unpresented explanations are baked into the decision-making process. For instance: “A person with a gun pointed at him probably feels fear.” “A person who utters racial epithets is more likely to have racially biased attitudes.” “A person with a financial interest in the outcome of a case is more likely than a disinterested witness to shade his testimony.” These are a few examples of the innumerable generalizations about the world that we expect—indeed, count on—jurors to bring to the fact-finding process.115 Though expected, and indeed necessary, the use of such generalizations creates the likelihood that the jury will develop and consider unpresented explanations and assign them a value greater than zero.

An additional difficulty facing IBE’s account of the burden of proof is the legal rule that the plaintiff does not automatically win even when the defendant offers no affirmative contrary explanation.116 Juries are not required to believe even uncontested evidence,117 and defense cases often consist of attacks on the credibility of the plaintiff’s witnesses, disbelief of whom can lead to the failure of proof on an essential element. Consider a product liability case in which the plaintiff’s explanatory story is that he suffered a stroke caused by taking the defendant’s anti-cholesterol drug Reactin. The defendant does not offer an alternative “story,” explanation, or even argument, of how some causal agent or agents other than Reactin—such as smoking, high-stress lifestyle, alcohol consumption, or what

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114. See Pardo, supra note 84, at 552.
117. See id.
have you—probably caused the stroke. Nor does the defendant even claim that Reactin did not cause the stroke. Instead, the defendant’s case is that the plaintiff’s experts did not use a reliable methodology in concluding that Reactin probably did cause the stroke. The jury finds for the defendant—as it is entitled to do—because it disbelieves the plaintiff’s expert testimony. This example, based loosely on *Daubert v. Merrill Dow Pharmaceuticals, Inc.*, 118 is not an outlier, nor is it specific to expert witness admissibility issues or summary judgment. Although in *Daubert*, the Ninth Circuit on remand excluded a key expert witness and granted summary judgment, 119 it remains true that even if the plaintiff’s expert testimony had been admitted and the case gone to trial, the jury would have been entitled to disbelieve the plaintiff’s expert based on the testimony of the defense experts and find for the defendant accordingly.120 This scenario represents a not-uncommon pattern in disputed-cause cases, in which the defendant asserts “we simply don’t know how the plaintiff was harmed.”

To call this defense the “best explanation” of the evidence is simply to finesse the definition of “explanation.” The plaintiff’s stroke is undisputed; the defendant’s negation of causation is no explanation of how the plaintiff’s stroke occurred in any meaningful sense of the term “explanation.” It jars with our intuitive sense of what “explanation” means. But we need not rely on intuition. It cannot be what IBE means by “explanation,” even one limited to the causation issue rather than the whole case. By saying “we cannot know the cause,” the defense in effect asserts the sum of all possible alternative negating explanations. Rather than the IBE formula of comparing \( Pr(A) \) and \( Pr(X) \), the jury is asked to compare \( Pr(A) \) to \( Pr(\text{not-}A) \).121 This is simply a reversion to the standard probability model. While there are a handful of statements in the IBE literature suggesting that the jury can consider and aggregate all possible not-

119. *Daubert v. Merrell Dow Pharm., Inc.*, 43 F.3d 1311, 1322 (9th Cir. 1995). There, the Ninth Circuit ultimately held that no reasonable jury could find for the plaintiff based on the defense experts’ refutation of the plaintiffs’ experts. *Id.* at 1320-22. But that famous case name should not send us down a tangential road of expert admissibility issues. The point is that the defendant successfully defended the case without presenting any sort of explanatory narrative of causation. See *id.* at 1315.
120. See supra note 73.
121. See supra note 72 and accompanying text.
A scenarios into a best explanation, IBE is not the best explanation of this situation. The standard probability formula is, because the defense case put the entire probability space of causation on the table, and the jury found that \( \Pr(A) \) was not greater than 0.5.

Allen’s more recent iterations of Explanatory theory in collaboration with Pardo attempt to shore up some of these descriptive shortcomings by changing the theory to permit fact-finders to consider unpresented explanations. This effort to be more catholic in explaining formerly ill-fitting cases gains in descriptive accuracy, but only at the cost of losing analytical crispness in distinguishing itself from the standard \( \Pr(A) > 0.5 \) account. The more unpresented hypotheses we permit the jury to consider, the closer the sum of the presented and unpresented hypotheses gets to 1.0, and the less analytical benefit is derived from the IBE account in relation to the standard probability account of \( \Pr(A) > 0.5 \). The extreme example would be the Reactin (Daubert) fact pattern, in which the defendant relied on attacking the credibility of key plaintiff witnesses rather than offering any sort of explanation of what happened. Again, that defense invokes the infinity of unpresented alternative explanations of plaintiff’s injury.

To be fair, there is an unresolved “speculation problem” in evidence law as a whole—there is no clear, consistent, or coherent approach to when a jury can and cannot consider unpresented hypotheses. Despite the common instruction to jurors not to “speculate,” the above examples show that they are often supposed to do just that. IBE can hardly be faulted for failing to solve this problem, which evidence law has yet even to recognize. Nevertheless, IBE’s ambiguity about unpresented explanations undercuts its probabilistic solution to the conjunction paradox. IBE either increases the risk of erroneous decision; bars defendants from presenting disjunctive defenses, contrary to existing law; or allows use of

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122. See, e.g., Pardo, supra note 84, at 599 (“General or disjunctive explanations may still be the best available explanation.” (emphasis added)). “General explanations”? But see Pardo & Allen, supra note 3, at 251 (“[T]he parties’ explicit theories at trial help to sort potential explanations.” (emphasis added)).

123. To a vaguely defined extent. Without sufficient elaboration, they say that “fact-finders are not limited to the potential explanations explicitly put forward by the parties, but may construct their own.” Pardo & Allen, supra note 3, at 234.


125. See supra note 113 and accompanying text.
unpresented explanations, thereby converging with the standard $Pr(A) > 0.5$ account of burden of proof.126

D. Conclusion: Are the Cures Worse than the Disease?

To sum up, each of the proposed solutions to the conjunction problem are workarounds that drop one or more assumptions from the 5-assumption model by which the conjunction paradox is created. Cohen and Clermont drop assumptions 1 and 2 by replacing the standard rules of probability with something else.127 Expositors of the element approach drop assumption 5, the “whole claim” condition.128 And proponents of IBE alter assumption 5 by dropping the $Pr(A) > 0.5$ requirement, and perhaps dismissing probability theory (assumptions 2 and 3) as well.129

It is not our aim to argue that any of the foregoing theories should be rejected as theories of justified belief in the litigation system. What we do claim, however, is that each of the theories has explanatory problems and complexities that impose potential costs if they were adopted in place of what we take to be the current prevailing understanding: that the burdens of proof are meant to be predetermined probability thresholds. Changing or dispensing with the axioms of probability theory, or dropping the requirement that a claimant prove his claim to a fixed probability threshold may require changing analytical or normative commitments whose nature remains to be more fully explored. They may also require changes to existing legal rules. For example, despite the claim by IBE proponents that their explanatory theory tracks existing trial rules and practice better than the standard $Pr(A) > 0.5$ account, the

126. Edward Cheng’s interesting attempt at solving the conjunction problem is also worth noting. Cheng argues that the conjunction problem is solved by recasting the parties’ claims as “probability ratios,” in which the plaintiff wins so long as the probability of his case is greater than that of the defendant’s case. See Cheng, supra note 10, at 1259-62. We have not discussed Cheng’s argument at length in the text here, because we read this argument as in essence recasting the probabilistic solution of IBE in probability formulas: because the plaintiff can win without showing $Pr(A) > 0.5$, this permits plaintiff to win at least some claims falling into the probability gap. It is subject to the same critique that we have made about the IBE probabilistic solution. Curiously, IBE proponents have rejected Cheng’s helping hand. See Allen & Stein, supra note 1, at 594-99 (attempting to refute Cheng’s argument).

127. See supra Part II.A.

128. See supra Part II.B.

129. See supra Part II.C.
plurality of jurisdictions instruct their juries in terms consistent with the $\Pr(A) > 0.5$ account, and a smaller minority state that the burden of proof is met by the stronger of the parties’ cases.\textsuperscript{130} 

In sum, there may be a large baby in the conjunction problem bathwater. Perhaps these changes are worth making for various reasons—but, as we will argue, solving the conjunction problem should not be one of those reasons.

III. Narrowing the “Probability Gap”: The Multiplication Rule for Dependent Events

In this Part, we examine assumption 3' of the conjunction paradox: that the overall probability of the claim is the product of the unconditional probabilities of the conjoint events. This assumption presupposes that the elements of claims are probabilistically independent events, requiring application of the multiplication rule for independent events. At least some evidence scholars in the past have argued that this assumption is wrong, at least some of the time; that the different multiplication rule for dependent events should be applied to conjoint elements of claims; and that doing so might resolve, or at least mitigate, the conjunction problem.\textsuperscript{131} But that argument seems to have dropped out of the discussion of the conjunction problem. Today, the conventional presentation of the conjunction paradox applies the multiplication rule for probabilistically independent events, as we noted above in assumption 3'.\textsuperscript{132}

\begin{itemize}
\item \textsuperscript{130} Compare Judicial Council of California Civil Jury Instructions § 200 (Judicial Council of Cal. Advisory Comm. on Civil Jury Instructions 2016) (“A party must persuade you, by the evidence presented in court, that what he or she is required to prove is more likely to be true than not true.”), with Revised Arizona Jury Instructions—Civil, Standard 2 (Civil Jury Instructions Comm. of the State Bar of Ariz. 5th ed. 2015) (“On any claim, the party who has the burden of proof must persuade you, by the evidence, that the claim is more probably true than not true. This means that the evidence that favors that party outweighs the opposing evidence.” (emphasis added)). The California instruction states the $\Pr(A) > 0.5$ standard in plain language. The Arizona instruction states that the better explanation meets the burden of proof. Fewer than 40% of the jurisdictions we surveyed adopted the latter comparative standard advocated by IBE. See David S. Schwartz & Elliott Sober, Appendix: Conjunction-Problem v. Non-Conjunction-Problem Jurisdictions, 59 Wm. & Mary L. Rev. Online 33 (2017).
\item \textsuperscript{131} See Dawid, supra note 39, at 92 (acknowledging scenarios in which elements may not be fully independent); Hamer, supra note 20, at 527-28 (same).
\item \textsuperscript{132} See Clermont, supra note 6, at 1108 (noting that “disputed elements might not be really independent” in practice, but using the multiplication rule for independent events
\end{itemize}
To be sure, probabilistic dependence fails to eliminate the conjunction problem mathematically. Because the probabilities of the elements are less than 1 under either product rule, the conjoint probability of elements is always less than the original individual probabilities. Given that probabilistic dependence does not eliminate the conjunction problem, many scholars are content to present the problem using the wrong multiplication rule—for independent events—to avoid what they see as undue presentational complexity.

Nevertheless, we believe that probabilistic dependence should be restored to the conversation. In this Part we argue that, since most elements of legal claims are probabilistically dependent, the multiplication rule for dependent events provides the proper analysis for the conjunction problem. The conditional probabilities required to be used in the multiplication rule for dependent events will typically have higher values than their unconditional counterparts. Since their values will still be less than 1.0, their product will still be diminished by adding more elements, and thus the probability gap is not eliminated. But while probabilistic dependence does not by itself eliminate the conjunction paradox, it does substantially narrow the “probability gap” that makes the conjunction paradox seem like a big problem.

A. Probabilistic Dependence and Independence

1. Formal Definitions and Multiplication Rules

Probabilistic independence and dependence are formally defined in probability theory as follows. Two events, $A$ and $B$, are probabilistically independent if and only if $Pr(A \mid B) = Pr(A)$. That is to
say, the probability of $A$ given $B$ (or $A$ conditional on the truth of $B$) equals the unconditional probability of $A$. In plain English, $A$ and $B$ are probabilistically independent if the occurrence of event $B$ has no effect on the probability of the occurrence of event $A$. Probabilistic independence can be defined equivalently as $\Pr(A \mid B) = \Pr(A \mid \neg B)$, because the probability of $A$ is the same whether $B$ is true or false.\footnote{See id. at 62.} The presence of $B$ in the factual mix does not affect the probability of $A$. It follows that events $A$ and $B$ are probabilistically dependent if $\Pr(A \mid B) \neq \Pr(A)$. In plain English, $A$ and $B$ are dependent events if the occurrence of $B$ affects the probability of the occurrence of event $A$.

2. Logical v. Probabilistic Independence

There is a significant difference between probabilistic independence, on the one hand, and logical independence on the other. Probabilistic independence has been defined above. Logical independence is a related but different concept. “Propositions $X$ and $Y$ are logically independent precisely when all four conjunctions of the form $\pm X \& \pm Y$ are logically possible (i.e., non-contradictory).”\footnote{E LLIOTT SOBER, OCKHAM’S RAZORS: A USER’S MANUAL 67 (2015). These are $X$ and $Y$, $X$ and $\neg Y$, $\neg X$ and $Y$, $\neg X$ and $\neg Y$.} The statements “it is raining’ and ‘you are carrying an umbrella’ are logically independent of each other”—you may be carrying an umbrella, or not, whether it is raining or not.\footnote{Id.} But “if you follow the advice of accurate weather forecasts, these two propositions will be probabilistically dependent on each other.”\footnote{Id.} Moreover, if the two propositions in question “are neither tautologies nor contradictions,” then probabilistic independence implies logical independence, but not the converse.\footnote{Id.}

Thus, logically independent propositions may be probabilistically dependent, as in the umbrella/raining example above. This distinction may have confused some evidence theorists who, mistaking logical independence for probabilistic independence, seem to have greatly overstated the presence of independent elements in legal
claims. Consider that two logically independent events that seem “clearly” to be probabilistically independent of each other may turn out to be probabilistically dependent once additional facts are brought to bear. Suppose event A is “the Denver Broncos win the Super Bowl” and event B is “the Democratic candidate wins the next presidential election.” Viewed one way, these events seem unrelated and independent. But suppose a Broncos Super Bowl victory makes a significant number of Colorado voters more satisfied with the state of the world, causing them to vote to reelect the Democratic incumbent, swinging a key state into the Democratic column. In both the Broncos/Democrats and the umbrella/raining examples, incorporated background facts render the logically independent propositions probabilistically dependent.

3. The Multiplication Rules

In basic probability theory, the probability of two events A and B occurring conjointly is their product: \( Pr(A \& B) = Pr(A \mid B) \cdot Pr(B) \).\(^{141}\) That is to say, the probability of A and B equals the conditional probability of A given B times the unconditional probability of B. This is the general rule, which is true whether A and B are independent or dependent. If they are independent, the rule simplifies to \( Pr(A \& B) = Pr(A) \cdot Pr(B) \). The simplification is made possible because independence means that \( Pr(A \mid B) = Pr(A) \).\(^{142}\)

When the question is the joint probability of three events, A, B, and C, the probability of the conjoint occurrence is expressed by \( Pr(A \& B \& C) = Pr(A \mid B \& C) \cdot Pr(B \& C) \). This is the same formulation of the original multiplication rule, in which the conjunction B & C has taken the place of B in the original version. But note also that \( Pr(A \& B \& C) = Pr(A \mid B \& C) \cdot Pr(B \& C) = Pr(A \mid B \& C) \cdot Pr(B \mid C) \cdot Pr(C) \).

This tells us that, where A, B, and C are mutually dependent events, only the last probability is unconditional when using the multiplication rule to determine their joint probability. But where A, B, and C are independent, the conditional probabilities drop out: \( Pr(A \& B \& C) = Pr(A) \cdot Pr(B) \cdot Pr(C) \).

\(^{141}\) Hacking, supra note 8, at 59.
\(^{142}\) Id. at 60.
B. The Conjunction Paradox and the Assumption of Independence

By treating the multiplication rule for independent events as a stand-in for the multiplication rule for dependent events, evidence scholarship may have underestimated the impact of probabilistic dependence on the conjunction problem in two ways. First, as we will explain further in the next Section, conjunction problem scholarship tends to underestimate the degree of probabilistic dependence of the elements. For example, the existence of a duty of care and the breach of that duty are often presented as two elements, rather than a single element.\(^{143}\) In that case, the existence of a duty given its breach will be extremely high, approaching 1.0.\(^{144}\)

Second, the scholarship underestimates the pervasiveness of dependence of the elements.\(^{145}\) In contrast to statements treating dependence as the exception, it is, we believe, the norm. Here, it is possible that at least some scholars have confused probabilistic independence with logical independence. Consider a two element claim: \(X\) is the defendant’s negligent act toward the plaintiff, and \(Y\) is the causation of damages to the plaintiff by the conduct. The two are logically independent, because all four conjunctions of the form \(\pm X \& \pm Y\) are logically possible (that is, noncontradictory). It is possible for the defendant to have breached a duty of care without causing harm to the plaintiff, or the other way around. But the two elements are probabilistically dependent: if the defendant did in fact act negligently toward the plaintiff, the probability that the defendant caused injury to the plaintiff is higher than it would be without the negligence.\(^{146}\)

Although identified by the law as a set of facts that “must” be proven probably true for a claimant to meet his burden of persuasion, and expressed at a relatively high level of generality, the elements are nevertheless “facts of the case.” Intuitively, we should expect the elements to be as interrelated as any other facts of the case. While the intuitive idea of interrelatedness is not identical to probabilistic dependence, it is close enough to make one wonder why

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144. See infra Part III.C.1.
145. See supra notes 132-34 and accompanying text.
146. See infra Part III.C.
it would be assumed that elements of a claim are probabilistically independent. For example, as will be explained further below, to say that a defendant’s negligent conduct is probabilistically independent of the plaintiff’s damages implies that that negligent conduct has no causal or evidentiary bearing on damages.\footnote{See infra Part III.C.} Assuming probabilistic independence as a starting point means that the plaintiff has lost the claim at the outset. As Judge Cardozo famously put it in *Palsgraf v. Long Island Railroad*, “negligence in the air, so to speak, will not do.”\footnote{Palsgraf v. Long Island R.R., 162 N.E. 99, 99 (N.Y. 1928) (quoting Frederick Pollock, The Law of Torts 455 (11th ed. 1920)).}

The counterintuitive nature of assuming litigated facts to be independent is even more pointed when we think about the actual presentation of cases. The law requires a claimant to present a detailed narrative in support of his claim that includes far more case-specific factual assertions than the relatively small set of facts comprising the “essential ... elements.”\footnote{See David S. Schwartz, A Foundation Theory of Evidence, 100 Geo. L.J. 95, 129 (2011); cf. Ashcroft v. Iqbal, 556 U.S. 662, 678-79 (2009) (explaining that an adequately pleaded cause of action must contain sufficient detail to show more than a possibility of entitlement to relief).} While the elements range between two and ten for most claims, the number of detailed factual assertions encompassed in the narrative may number in the dozens or even hundreds.\footnote{See Schwartz, supra note 149, at 129-32.} Each one of those factual assertions will have its own probability, and the probability of the overall narrative will be the product of these probabilities. If all these case-specific fact assertions were probabilistically independent of each other, we would have a far bigger conjunction problem on our hands than has been recognized. That would mean, paradoxically, that the more detailed the narrative presented in support of claim $X$, the less probable it is that $X$ is true! The conjunction paradox would thus be made into an analogue of Zeno’s paradox of motion: no claim can ever be logically proven unless each component fact (of which there are an infinite number) is proven to a virtual certainty.\footnote{This is a matter of axiomatic probability theory and basic arithmetic. As we increase the number of independent events, the conjoint probability approaches zero: the probability of ten independent events of 80% probability occurring conjointly is less than 11%. To counteract this tendency and thus maintain a relatively high level of probability—even the greater than 50% threshold required to meet the civil burden of persuasion—the probability...}
The solution to this paradox requires recognizing that detailed facts that make up a claimant’s narrative of guilt or liability tend to be probabilistically dependent. In fact, by virtue of the requirement of relevance, all facts entailed by the offering party must change the probability of other facts; the narrative is supposed to be a network of facts relevant to one another, and irrelevant facts are to be excluded. The probability of evidentiary fact \( A \) given evidentiary fact \( B \) is greater than without \( B: \Pr(A \mid B) > \Pr(A) \). Note that this statement, which formularizes the Federal Rules of Evidence (FRE) 401 standard of relevance, defeats independence, which is defined as \( \Pr(A \mid B) = \Pr(A) \). By definition, two facts relevant to one another are probabilistically dependent.

It is also important to note that the conditional probabilities that we must use in calculating the joint probabilities of these facts can be much higher than the unconditional probabilities. The relevance and factual interrelatedness of a persuasive story of liability is likely to mean that all but the weakest link will have high conditional probabilities. Consider a shaky eyewitness identification that is corroborated by strong circumstantial evidence that the defendant committed the crime. The unconditional probability of the eyewitness’s testimony might be around 0.5, but its conditional probability given the circumstantial evidence might approach 1.

C. The Probabilistic Dependence of Most Elements of Litigated Claims

Elements of claims are not different from evidentiary facts for purposes of determining dependence. Elements differ from evidentiary facts in the sense that they are stated generically when describing the substantive law. Nevertheless, elements of claims are factual propositions, and they are proven in a specific case in terms of case-specific evidentiary facts. In operation, elements share with

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of the individual events must get closer and closer to 1 (that is, 100% certainty). In order for an infinite number of multiplicands to multiply out to any number that does not asymptotically approach zero, the multiplicands must asymptotically approach 1. For example, for the conjoint probability of four equally probable events to exceed 0.5, the probability of each must be more than 84%; for ten such events, 93%; and for one-hundred such events, 99%. The fourth root of 0.5 is .841; the tenth root is .933; the hundredth root is .993.

152. See Fed. R. Evid. 401.
every other factual proposition the linguistic and logical property that they can be subdivided into numerous subpropositions.

The simplest illustration of this point is found in the variable formulations of negligence claims in the conjunction problem literature. A negligence claim might be stated as a single factual proposition that “the defendant negligently injured the plaintiff.” But the substantive law typically divides this proposition into four parts: (1) the defendant owed the plaintiff a duty of care; (2) the defendant breached that duty of care; (3) the plaintiff suffered damages; and (4) the damages were caused by the defendant’s breach. This is the standard hornbook litany “duty, breach, causation, damages.” Each of these propositions is an “event” within the meaning of probability theory, and can be assigned a probability. But we might combine “duty and breach” into a single element, “breach of duty” or “fault.” Likewise, causation and damages can naturally be combined into a single element. It is not surprising that discussions of the conjunction problem use three- or two-element formulations of negligence.

Yet these semantic and syntactical reconfigurations do not require different evidence, and thus can hardly change the probability of the whole. As Richard Friedman has explained, “Because the redivision of the claim [into more elements] has not altered its substance, the fact-finder’s assessment of the probability of the truth of the entire claim cannot have changed.” If there is a 0.6 probability that the defendant negligently injured the plaintiff, that probability is the same whether the claim is sent to the jury with the foregoing four-, three-, or two-element instruction on negligence. It is true that the four-element version of the negligence instruction would call for the product of four multiplicands, and the two-element version would call for only two; and the multiplicands in the four-element version would have to reflect higher individual proba-

153. See, e.g., Spottswood, supra note 6, at 274-75.
155. See id. (describing three- and four-element versions of negligence); see also KERTON ET AL., supra note 143, § 30 (describing a four-element version).
156. See, e.g., Cheng, supra note 10, at 1256 (illustrating a “conventional negligence claim” consisting of three elements); Nance, supra note 20, at 1566 (illustrating a two-element negligence claim).
157. Friedman, supra note 35, at 283.
bilities than those in the two-element version to multiply out to 0.6. What is not true, however, is the commonplace assertion that the four-element negligence claim is therefore harder for the plaintiff to prove than the two-element version. Where the profusion of elements is merely linguistic, and does not add previously unneeded facts to prove the case, the multiplication rule creates no practical problem of proof.

Even when the function of subdividing a claim into evidence is to guarantee a certain degree of factual detail of the evidence—in essence to require more evidence than in a less elaborate statement of elements—much of the seeming resultant “probability gap” will be obviated by probabilistic dependence. An examination of large numbers of jury instructions shows that most elements of most claims are probabilistically dependent. This can occur for at least two related reasons: the elements entail one another, or the elements are relevant to one another in the FRE 401 sense, as we now explain.

1. Overlapping or Shared Elements

Many claims are defined in terms of elements that overlap: they entail other elements or have shared propositions. Consider the first two elements of the four-element version of negligence:

158. See, e.g., Cheng, supra note 10, at 1257 (“[W]ith each additional element, the plaintiff finds it increasingly difficult to win.”). We should be clear: what we are talking about are linguistic formulations that subdivide the definition of a claim requiring a certain amount of evidence to prove a certain overall probability. There will be cases where an additional element does make the plaintiff’s claim more difficult to win. In *Virginia v. Black*, for example, the Supreme Court held that a statute making it a crime (1) to burn a cross (2) on the property of another was unconstitutional unless a third element was added: (3) with intent to intimidate. See 538 U.S. 343, 347-48, 362-64 (2003). There, the Court’s purpose was specifically to make the claimant’s case more difficult to prove—the additional element required additional evidence. See id. That is not an instance of linguistic reformulation, but a change in the substantive evidentiary requirements.

Nor are we talking about the potential for making the claimant’s case more difficult because an increase in elements might increase the likelihood of jury confusion. That may affect probabilities, but not in the sense the conjunction problem contemplates. See supra note 27.

159. See infra Part IV.C.3.
(A) Defendant owed plaintiff a duty of care.

(B) Defendant breached that duty of care.160

Breach and duty are hardly independent: a duty of care may exist independently of a particular breach, but a breach cannot exist without a duty; “breach” is shorthand for “breach of a duty of care.”161 So the probability of duty given breach is very close to 1.0. A full statement of the propositions entailed in “duty” and “breach” would be as follows:

Proposition A: Defendant owed plaintiff a duty of care.

Proposition B: Defendant owed plaintiff a duty of care and breached that duty.

The second element of negligence entails the first. This implies probabilistic dependence in all cases where the facts are disputed. Note, first of all, that propositions A and B are not logically independent: it is not the case that all four conjunctions of the form ±A and ±B are noncontradictory: not-A contradicts B. A and B cannot be probabilistically independent, since logical independence is a necessary condition of probabilistic independence.162 Conversely, if two events are logically dependent, they are probabilistically dependent.163

We can also demonstrate the probabilistic dependence of A and B by proof, but we have to spell out the significance of “disputed facts.” Standard formulations of modern probability theory entail that probabilities of 0 and 1 are “sticky,” meaning that they are not amenable to change in light of new evidence.164 Thus, while probabilities of 0 and 1 are assigned to logical contradictions and tautologies, respectively, they are generally excluded from real-world probability assessments.165 The litigation system embraces

160. See Keeton et al., supra note 143, § 30.
161. See Fishman & McKenna, supra note 154, § 51:17.
162. See supra Part III.A.2.
163. See supra Part III.A.2.
164. See Sober, supra note 3, at 152.
165. See id.
this understanding. An element of a claim may be assigned a probability of 0 or 1 by a court (judgment as a matter of law), or may be assigned a probability of 1 by the parties (a stipulated fact). But these are defined as “undisputed facts.” Factual disputes assigned to the jury are considered inherently uncertain: they are not expected to be resolved as 100% true or 100% false, and juries are expected to assess probabilities based on evidence. Accordingly, disputed facts will, by assumption, have probabilities greater than zero but less than one. The proof is now quite simple:

If \( B \) entails \( A \), then \( Pr(A | B) = 1 \) [by axiom], \( Pr(A) < 1 \) [by assumption], therefore \( Pr(A | B) > Pr(A) \), which shows probabilistic dependence.

In the negligence example, it will be seen that the nesting or entailment of elements into one another does not stop with breach. The next element is causation, but “cause” in the sentence defining negligence is a predicate that is inchoate—its meaning cannot be fixed by a trier of fact—without a subject and an object. The subject is proposition \( B \), and the object is proposition \( D \), damages.

Proposition \( C \): The breach of the duty of care caused an accident resulting in damages.

Proposition \( D \): The plaintiff suffered damages from the breach of the duty of care.

Causation in the air, so to speak, will not do either. Nor will the one word “damages,” for element \( D \) is not satisfied by damages coming from other causes.

166. See HACKING, supra note 8, at 60.
167. Note that if \( B \) entails \( A \), the two could be independent by assigning \( A \) or \( B \) some combination of probabilities of 1 or 0. But a 0 probability ends the case under either multiplication rule, while a 1 probability has no numerical impact on the product of the remaining elements. Where the separate probabilities of the elements matter, elements that overlap or entail prior elements are probabilistically dependent.
2. Mutually Relevant Elements

As we showed above, the definition of probabilistic dependence is identical to the definition of relevance in the FRE. Evidence $B$ is relevant to consequential fact $A$ if $B$ changes the probability of $A$. In other words, the definition of relevant evidence, $Pr(A | B) \neq Pr(A)$, contradicts the definition of probabilistic independence, $Pr(A | B) = Pr(A)$. This applies equally to elements of the case, which are, after all, factual propositions. Any elements connected to one another as potentially relevant evidence are probabilistically dependent.

A recurring illustration of this is the concept of causation of damages, or more generally, “harm.” The elements of virtually all civil cases include a requirement that the defendant’s conduct (or some agent for whose conduct the defendant is legally responsible) caused some type of harm to the plaintiff. 168 The structure of this claim asserts that conduct $A$ caused harm $B$. Generally speaking, factual propositions of this “$A$ causes $B$” structure imply that $A$ and $B$ are probabilistically dependent. 169 In the litigation context, it is easy to see that a civil wrong, such as a tort or breach of contract, increases the probability that the recipient of that wrongful conduct suffered damages. The reverse is also true: where a person has suffered injury, the probability of misconduct by an identifiable perpetrator acting on that person is at least slightly increased. This is intuitive: breaches of duties of care tend to cause injuries, breaches of contract tend to cause financial losses, etc.

It would be difficult to identify many civil or criminal claims that do not have this type of probabilistic dependence as to some of the elements at least. In an armed robbery case, the elements might be (1) the taking of property by means of, (2) force or threat, (3) while using a weapon. 170 Facts showing the threat will also tend to show the use of the weapon, and vice versa. Facts showing the taking of property will be relevant to show the threat, and vice versa. In all such instances where the elements have a tendency to assist in

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168. See, e.g., supra Part III.C.I.
169. We say “generally speaking” because examples can be constructed in which this condition fails to be satisfied. See, e.g., Austin Frakt, Causation Without Correlation Is Possible, INCIDENTAL ECONOMIST (Dec. 16, 2009, 4:00 AM), http://theincidentaleconomist.com/wordpress/causation-without-correlation-is-possible/ [https://perma.cc/YZ9L-P2X4].
proving one another—to make one another more probable—the elements of the claim are probabilistically dependent.

D. Applying the Proper Multiplication Rule

It is easy to see how correcting the error of assumption 3′ can significantly narrow the appearance of a “probability gap.” In the multiplication rule for dependent events, only the last probability is unconditional.171 Thus, for example, with three interdependent events, essential elements \( A, B \), and \( C \), the following formula applies:

\[
\Pr(A \& B \& C) = \Pr(A \mid B \& C) \cdot \Pr(B \& C) = \Pr(A \mid B \& C) \cdot \Pr(B \mid C) \cdot \Pr(C).
\]

The conditional probability of one element given another will be higher than the unconditional probability of that element and as a practical matter is often quite high. It may even be close to 1. Only the last element will be represented by an unconditional probability. Consider the two-element version of negligence: if the conditional probability of fault given causation were 95%, an unconditional probability for causation of just over 52.63% is all that is arithmetically required for the product to exceed 0.5.

To repeat, we do not claim to have solved or eliminated the conjunction paradox. The probability of the whole claim will always be less than the lowest unconditional probability.172 But we do believe

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171. An extra unconditional probability will have to be multiplied in cases where there is a probabilistically independent element. We suggest that most claims involve probabilistically dependent elements, though we don’t claim to have exhaustively canvassed the entire world of civil and criminal claims. We are aware of one recurring category of claims that include a probabilistically independent element: certain status offenses and perhaps claims with a jurisdictional element. For example, the two elements of the crime of felon in possession of a gun are (1) possession of a gun, and (2) by a person convicted of a felony. See 79 AM. JUR. 2D Weapons and Firearms § 26 (2017). These appear to be probabilistically independent. The same point might be raised about the jurisdictional element in the federal version of this crime. See 18 U.S.C. § 922(g) (2012) (“It shall be unlawful for any person ... who has been convicted in any court of, a crime punishable by imprisonment for a term exceeding one year ... [to] possess in or affecting commerce, any firearm.”). Vicarious liability may be another example: proof that a tortfeasor was employed by the defendant seems to be probabilistically independent of the elements of negligence, causation, and damages.

These exceptions do not significantly affect the main point. The majority of litigated claims do not involve these elements, and many statuses, such as a felony conviction, are easily proven to a high degree of probability. And in a vicarious liability case where the status and the tort both carry high degrees of doubt falling just shy of 0.5, one has to wonder why it is problematic to find for the defendant.

172. See, e.g., Cheng, supra note 10, at 1257. We say the “lowest unconditional probability” instead of the unconditional probability of the last element because probability theory is
we have shown that the conventional presentation of the conjunction paradox greatly overstates the appearance of a “probability gap.”

IV. THE NATURE OF THE JURY FUNCTION

The probability gap is not a theoretical problem, or a paradox. It is merely an atmospheric that sustains an illogical intuition that separate probabilities of elements should not have to be higher than the probability of the overall claim. What makes the conjunction problem paradoxical is the conflict between assumptions 4b and 5.

4b. Proving that the probability of each separate element exceeds the probability threshold established by the applicable burden of proof is a sufficient condition for the claimant to win the claim.

5. If the probability of the overall claim is greater than the probability threshold established by the applicable burden of proof, the claimant should win the claim; otherwise she should lose.

Assumption 4b, whatever its source as an idea, appears to be interjected into trial decision-making by jury instructions.\textsuperscript{173} In this Part we argue that the legal foundation of assumption 4b is ambiguous. Few jurisdictions in the United States instruct their jurors to find for the claimant based on meeting an “each element” condition.\textsuperscript{174} The language of most jury instructions can be read to harmonize assumptions 4 and 5 by incorporating assumption 4a rather than 4b—proving each element to the probability threshold is a necessary but not sufficient condition for finding guilt or liability.\textsuperscript{175} The purpose of isolating the elements may not be to require fact-finders to determine conditional probabilities, or to multiply indifferent to the order in which the elements are stated. The conditional and unconditional probabilities must multiply out to the same product whichever order they appear in the formula. We explain this further in Part IV.

\textsuperscript{173} See, e.g., Allen & Jehl, supra note 15, at 897-904.

\textsuperscript{174} See infra Part IV.C.3.

\textsuperscript{175} See infra Part IV.C.3.
conditional or unconditional ones, but rather to run a simple test on
the probability of the overall claim—a test we call an "entailment
check." However, special verdict forms introduce an additional layer
of ambiguity into the mix.

A. Why Do We Have “Each Element” Jury Instructions?

At the end of the day, a claimant is required to prove a conjunc-
tion, a narrative consisting of numerous facts at some level of detail.
The claim is in fact a unique narrative of liability—unique not in
the sense of “unusual,” but in the sense of particular. The elements
of claims are generic, not unique. When we ask a claimant to “prove
negligence,” we do not mean that the claimant should demonstrate
that the legal system’s general idea of negligence is defined as
“breach of a duty of care, causing damages.” Rather, the plaintiff is
to prove a unique instance in which negligence happened to him.
The claim is manifestly the focal point of the inquiry, not the
elements. Surely, whatever objections may be made to IBE, this is
one insight IBE expositors have gotten right: the claim is first and
foremost a case-specific explanation incorporating the legal ele-
ments, whose overall probability is determinative.176

So why do we have jury instructions that direct attention to “each
element”? The primary reason has to be to ensure “narrative fit”
between the claimant’s story and the implicit narrative built into
the substantive law.

The legal narrative is reduced to a checklist of “elements” ... that
must be present in order for legal consequences to be imposed as
a result of the particular occurrence. This “element” requirement
is not unlike a screenwriting rule that a “romantic comedy” must
have the elements “boy meets girl, they fall in love, they fall out
briefly, and they end up together when the boy makes a moving
speech after running or driving some great distance to find the
girl.”177

176. See supra notes 84-87 and accompanying text.
177. Schwartz, supra note 149, at 132. Aside from the regrettably hetero-centric quality of
the definition, have we misidentified the elements of romantic comedy? Maybe so. Perhaps
Butch Cassidy and the Sundance Kid fits the definition of a rom-com: two outlaws meet, fall
in love, start a relationship as train- and bank-robbing accomplices, have a falling out, recon-
cile and reunite—and die together in a hail of bullets. See BUTCH CASSIDY AND THE SUNDANCE
The jury instructions tell the jury to examine the elements of the case to determine whether the claimant has proven a claim that fits the negligence “genre” established by the substantive law.

B. The “Each Element” Instruction as an Entailment Check

Jury instructions are not directing juries to find for the claimant when the probability threshold has been reached for each element. Nor are they directing juries to multiply the probabilities they do find. What, then, is their function? Here, we elaborate on Nance’s insight that the “each element” condition (assumption 4a) functions “simply to remind the jury that a failure by the plaintiff to sufficiently prove any element of the plaintiff’s case will relieve the jury of further deliberations on the other elements.”

If a civil claim has essential elements A, B, and C, then the overall probability of the plaintiff’s case is formulated as:

\[ \Pr(A \& B \& C) = \Pr(A \mid B \& C) \cdot \Pr(B \& C) = \Pr(A \mid B \& C) \cdot \Pr(B \mid C) \cdot \Pr(C). \]

We have suggested that instructions do not ask the jury to determine these conditional probabilities, let alone multiply them. That seems to be asking a lot. As we have seen, even evidence scholars themselves have for many years shied away from doing so.

Kid (Twentieth Century Fox 1969). You could make a case—the film has some laughs in it.

This problem arises in the law all the time. The elements of murder might be defined as (1) the premeditated killing of another person (2) without justification. See 40 A M. JUR. 2D Homicide § 36 (2017). These two elements suggest a core or “garden variety” narrative that looks quite different from a case in which a doctor administers a lethal dose of morphine to a pain-stricken, terminally ill cancer patient. But the doctor could well be charged with murder. See T. Howard Stone & William J. Winslade, Physician-Assisted Suicide and Euthanasia in the United States, 16 J. LEGAL MED. 481, 484 (1995) (“A physician who commits euthanasia may be criminally charged with the patient’s homicide in virtually any state because the physician has purposely and directly caused the patient’s death.”). The uncertainty about the fit between the doctor’s specific-alleged-murder narrative and the core narrative of murder and its essential elements will be resolved either by a legal ruling or by a probabilistic jury decision. A judge might dismiss the case as a matter of law, in essence holding that the case-specific narrative does not include all the essential elements of murder. Or the judge might make a legal ruling that assisted suicide is not a justification in a case of murder. Or the judge might decline to define “justification,” and instead leave the jury to decide whether physician-assisted suicide is, beyond a reasonable doubt, an insufficient justification for a premeditated killing.

178. Nance, supra note 18, at 950.

179. See supra note 134 and accompanying text.
Let’s assume a case involving excessive force by a police officer, and assign each of the three elements a place in the above formula: $A = $ unreasonable force, $B = $ causation, and $C = $ compensable damages. If the legal system insisted on juries conducting a multiplication exercise to determine the probability of plaintiff’s excessive force claim, it would have to multiply: $Pr(A \mid B \& C) \cdot Pr(B \mid C) \cdot Pr(C)$. In words, we would be asking a jury to find and multiply the (conditional) probability of unreasonable force given causation and damages, the (conditional) probability of causation given damages, and the (unconditional) probability of damages.

But as a matter of probability theory, we have arbitrarily assigned elements to the letters $A$, $B$, and $C$ in the formula. The multiplication rule is indifferent to which event is assigned which place in the formula. We could as easily have formulated the probabilities in the claim as the conditional probability of damages given causation and unreasonable force, times the conditional probability of causation given unreasonable force, times the unconditional probability of unreasonable force. Perhaps we want the jury to do both, as well as the other possible combinations.

This aspect of the problem gives us a hint at its solution. The fact is that we do not actually expect the jury to find and multiply these probabilities. The “each element” jury instruction directs a quite different, more intuitive, and ultimately easier probabilistic operation. We call that operation an “entailment check.”

We can start with the intuition that unconditional probabilities are easier to describe and estimate than conditional ones. To be sure, that assumption has been debated, and we do not base it on psychometric authority. But this assumption seems to be widely shared by evidence scholars who have discussed the conjunction problem: hence, the marked tendency in the literature to present the conjunction problem in terms of unconditional probabilities to “simplify” matters.\(^{180}\) Certainly, it is true by definition that an unconditional probability is assessed in isolation from the probabilities of other events, which may simplify them in practice.

As noted above, the multiplication rule for probabilistically dependent events requires the finding of one unconditional probability; in our three-element example, this was $Pr(C)$. And, as noted

\(^{180}\) See supra note 134 and accompanying text.
above, we could run the formula three different times, assigning a different essential element to the place of $C$ each time. Axiomatically, the total probability $Pr(A & B & C) = Pr(C & B & A)$. This is because the two conjunctions are logically equivalent and the axioms of probability entail that logically equivalent propositions must have the same probability.\footnote{See Hacking, supra note 8, at 58; see also Friedman, supra note 35, at 283 (“Because the redivision of the claim [into more elements] has not altered its substance, the fact-finder’s assessment of the probability of the truth of the entire claim cannot have changed.”).} There are actually six different ordering combinations.\footnote{A-B-C, A-C-B, B-A-C, B-C-A, C-A-B, and C-B-A.} Running all six variations of the formula means that each element—unreasonable force, causation, and damages—will have its turn (two turns, in fact) occupying the $C$ position, the one assessed as an unconditional probability.

The “each element” instruction asks the jury to examine the elements for compliance with the doctrine of entailment (also known as the doctrine of logical consequence). As we have seen, seemingly simple statements can be shown to be equivalent to a conjunction.\footnote{See supra Part III.C.} The initial proposition is said to “entail” the conjuncts in the conjunction and cannot be more probable than any of the entailed components. If fact $B$ entails fact $A$, the probability of $B$ is less than or equal to the probability of $A$.\footnote{Hacking, supra note 8, at 60.} This is a corollary of the two multiplication rules. The factual assertion “he arrived at the airport on time” entails “he arrived at the airport,” and thus the probability that “he arrived at the airport on time” cannot be greater than the probability that “he arrived at the airport (at all).” By requiring “essential elements,” the substantive law tells us that a case-specific story of guilt or liability must entail case-specific versions of certain generic facts—for example, breach of a duty of care. By requiring that each element be proven by a preponderance of the evidence, the jury is instructed to check to make sure that each required fact is present at a level of certainty that meets the burden of persuasion. If any required fact falls below that level, then, under the doctrine of entailment, the entire story of guilt or liability falls below that level—since the entire story cannot be more probable than its least probable entailed fact.
Thus, the “each element” form of jury instruction fulfills two important functions, neither of which requires the jury to multiply probabilities or determine conditional probabilities of separate elements. First, as explained above, the listing of elements ensures that the facts deemed essential to a particular claim are all present.185 Second, the requirement that each element be proven to the burden-of-proof probability threshold provides a useful test on whether the overall claim meets the threshold. If one or more elements falls below the probabilistic floor, then the entire claim fails—according to the multiplication rule or its corollary, the doctrine of entailment.

True, the entailment check is a test that demonstrates only the failure of proof: passing the entailment check does not demonstrate that the claim as a whole has met the burden of proof. That requires either instructions to the jury to consider the probability of the overall “claim,” or perhaps in some jurisdictions reliance on jurors’ intuitions to consider whole claims. The entailment check embodied in jury instructions is merely an important, though imperfect probabilistic test of that overall finding.

In sum, the entailment check confirms the idea that meeting the probability threshold for the burden of proof for each element is a necessary but not sufficient condition for the claimant to win the claim. Pass the test, and the jury may find for the claimant. Fail the test, and the jury cannot find for the claimant.

C. What Do Jury Instructions Actually Say?

As noted above, it is not probability theory, but rather the set of decision rules given to fact-finders by the legal system, that creates the conjunction problem. This makes it important to attend to what jury instructions actually say. To that end, we have undertaken a survey of pattern jury instructions from fifty-five jurisdictions, both states and federal circuits, in civil and criminal cases. Our methodology is explained below,186 and the data are summarized in an Appendix to the Article.187 Overall, we found that jury instructions

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185. See Schwartz, supra note 149, at 132.
186. See infra Part IV.C.1.
replicated the conjunction problem in fewer than one-fourth of the jurisdictions in civil cases, and one-third in criminal cases.\textsuperscript{188} These jurisdictions replicated the conjunction problem by instructing juries to find for the claimant when the burden of proof has been met as to each element separately. But the majority of jurisdictions implied that juries should, or at least could, treat the satisfaction of the “each element” condition as necessary but not sufficient to find for the claimant. These jurisdictions are thus consistent with our “entailment check” explanation.\textsuperscript{189} However, even in these latter jurisdictions consistent with the entailment check explanation, the reliance on special verdict forms introduces a further ambiguity; special verdict forms can be understood to direct a finding for the claimant where the “each element” condition has been met.\textsuperscript{190}

It is important here to note the limits of our contentions. We are not claiming that jury instructions mostly “get it right,” or that they are for the most part free of errors in the way they present probabilistic reasoning. Nor are we claiming that jury instructions consistent with our entailment check explanation of the elements-claims relationship have been consciously designed to make jury decision-making conform to probability theory and the multiplication rule. Nance is undoubtedly correct when he surmises that few, if any, jury instructions have been written as self-conscious efforts to incorporate the multiplication rule for conjunctions.\textsuperscript{191} What we do claim is that the actual practice of jury instructions, as far as our survey reveals it, fails to demonstrate a systematic breakdown of jury decision-making stemming from a conjunction problem. Jury instructions can be presented in a way that obviates the conjunction problem, and many actually are. But we reserve a discussion of the normative question—whether jury instruction practice should be reformed to obviate any trace of the conjunction problem—for the last Section.

\textsuperscript{188} See infra Part IV.C.1.
\textsuperscript{189} See infra Part IV.C.3.
\textsuperscript{190} See infra Part IV.C.4.
\textsuperscript{191} See NANCE, supra note 20, at 77.
1. Methodology and Interpretations

In our survey, we examined state-by-state and federal circuit-by-circuit pattern jury instructions from fifty-five jurisdictions (forty-seven states and eight federal circuits) that were readily available from online sources. All the jurisdictions separate civil and criminal jury instructions, and some jurisdictions had only civil or only criminal instructions. This approach yielded data from forty-one states and six federal circuits in civil cases, and thirty-seven states and eight federal circuits for criminal cases. Although this data therefore has some obvious limitations, we judged that it was adequate to the present task, and we doubt whether there is sufficient additional information to be gleaned that would warrant the immense research effort involved in an exhaustive survey.

For each jurisdiction, we examined the instructions to detect the presence of the “each element” and “whole claim” conditions. We asked: did the instructions say that meeting the burden of proof as to each element was a necessary or sufficient condition, and did the instructions indicate a requirement that the entire claim meet the burden of proof? In each jurisdiction’s instructions, we looked in two places for this information. First, we examined the general instructions on the burden of proof, which typically address elements and claims generically. Second, we sampled substance-specific instructions, which list the elements of specific civil claims and criminal charges, to see whether these also gave directions about applying the burden of proof to elements and whole claims. Our

192. This count of forty-seven states includes the District of Columbia.
193. See Schwartz & Sober, supra note 130.
194. All data gathering involves choices and compromises, and this survey is no exception. States and federal circuits are missing from our survey either because they fail to issue pattern jury instructions, make their pattern instructions available only to members of the state bar, or require the payment of a fee to obtain them. Further, the reliance on pattern instructions itself represents a compromise of sorts in data gathering. Pattern jury instructions are not binding on trial courts in all jurisdictions, and there may be significant variation in practice as to the extent to which they are used in the exact form presented. Appellate decisions in a jurisdiction might provide a more accurate picture of what jury instructions are actually used—or they might not. Because not all jury instructions will find their way into appellate case law, and because appellate decisions are notoriously incomplete as descriptions of trial-level practice for a variety of reasons, even that monumental research undertaking is likely to be nonexhaustive.
195. See Schwartz & Sober, supra note 130.
analysis thus fills a gap in the existing debate over whether jury instructions replicate the conjunction problem: prior analyses did not systematically look at the linguistic interaction between the general and specific instructions, and thereby may have missed important information about the relationship between the burden of proof as to claims and elements.

Our inquiry shows a wide variation in how jurisdictions instruct juries on burdens of proof. But it confirms the conclusion in earlier scholarship based on more limited surveys that jury instructions do not for the most part reproduce the conjunction problem. We detected six distinct patterns in these instructions. Two patterns replicate the conjunction problem in that they direct juries to find for the claimant when the burden of proof has been met as to each separate element (assumption 4b). One of these two patterns replicates the conjunction problem explicitly, by including assumption 5 (instructing juries to base findings on whole claims) along with assumption 4b, while the other does so only implicitly, by including assumption 4b but omitting assumption 5.

The remaining four patterns, comprising the great majority of jurisdictions, fail to replicate the conjunction problem because they do not clearly include assumption 4b in their instructions. Several jurisdictions expressly incorporate assumptions 4a and 5: they instruct their juries to apply the burden of proof to both whole claims and elements, but do not require finding for the claimant when the burden of proof has been met as to the elements. A fourth pattern, which we refer to as “aggregate elements,” is found in jurisdictions that incorporate assumption 4b while implying—albeit ambiguously—assumption 5: they break down claims into elements and instruct their juries to find for claimants who meet the burden of proof on “all” the elements. This is the instructional pattern that Nance found to be ambiguous but consistent with what we are calling the entailment check. A fifth pattern incorporates assumption 4a only, instructing juries to apply the burden of proof to each separate element, without requiring a finding for the claimant where the “each element” condition has been met. This pattern is

196. See Levmore, supra note 65, at 724 & n.1; Nance, supra note 20, at 1571-72, 1572 n.69.
197. See infra Part IV.C.2.
198. See supra Part IV.B.
consistent with the entailment check if one assumes that juries
temporarily tend to decide whole claims unless instructed otherwise.
Finally, the sixth pattern treats claims holistically, in essence
incorporating assumption 5 while not including either assumption 4a
or 4b. In our view, this pattern fails to replicate the conjunction
problem, although it does not replicate the entailment check either.

The breakdown showing the number of jurisdictions for each of
these six patterns, separated by civil and criminal cases, is pre-
sented in Table 1. The bottom line is that the conjunction problem
appears in only eleven of forty-seven jurisdictions in civil cases, and
fifteen of forty-five jurisdictions in criminal cases.

Table 1. Conjunction-Problem v. Non-Conjunction-Problem
Jurisdictions

<table>
<thead>
<tr>
<th>Conjunction-Problem</th>
<th>Non-Conjunction-Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Claims &amp; Elements Only, (4b &amp; 5)</td>
</tr>
<tr>
<td>Civil</td>
<td>1</td>
</tr>
<tr>
<td>Criminal</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
</tr>
</tbody>
</table>

The distinction between the conjunction-problem and non-con-
junction-problem jurisdictions required us to make an interpretive
choice about a linguistic ambiguity that pervades the English lan-
guage. As linguists have observed, the terms “each,” “every,” “each
and every,” “any,” and “all” have overlapping meanings, and their
use in legal instruments is ambiguous in many contexts. 200
Nance pointed to this type of ambiguity in his essay launching the debate
over whether jury instructions in fact replicated the conjunction
problem. 201 According to Nance, instructions that require juries to

200. See 2 JOHN LYONS, SEMANTICS 454-59 (1977); see also Nance, supra note 18, at 949
(describing possible interpretations of a federal jury instruction).
201. See Nance, supra note 18, at 949.
find for the claimant who meets the burden of proof on “all” or even “every” element, could be read to mean that juries should base their findings on whole claims. Allen and Jehl subsequently disputed this point, arguing that the “each/every/all” ambiguity was cleared up by further instructions. Either juries are directed to find for the defendant if “any” element was not proven, or they are instructed to find for the claimant where each element had been proven to the probability threshold.

We have decided to split the difference between these two views. On the one hand, we agree with Nance that Allen and Jehl’s refutation is unpersuasive. Instructing juries to find for the defendant by the failure of proof on “any” element merely demonstrates that proving “each” element is a necessary condition, not that it is a sufficient condition. And instructing juries that they must find for the claimant when the burden of proof has been met for each element tells us nothing about whether “every” or “all” mean “each separate one” or “all together.” Nevertheless, the words “each” and “every” in most of the jury instructions strike us as more naturally understood to mean “each separate one,” rather than “all together,” and we have categorized the instructions accordingly. However, where an instruction indicates that a finding is based on meeting the burden of proof as to “all” elements, we have treated that instruction as suggestive of the “all together” interpretation.

2. Conjunction-Problem Jurisdictions

The conjunction problem arises expressly in jury instructions that state both that the claimant wins by proving each element to the probability threshold (assumption 4b), and that the claimant wins by proving his overall claim to the probability threshold (assumption 5). Because of the multiplication rule for conjunctions, such instructions, scrupulously followed, are likely to produce cases in which the “each element” and “whole claim” decision rules require conflicting results. Surprisingly, we found only one jurisdiction

202. See id. at 949-52.
203. See Allen & Jehl, supra note 15, at 897-904.
204. See id.
205. See Nance, supra note 20, at 1567.
206. See supra Part II.B.
employing such instructions in civil cases. The District of Columbia instructs that “[t]he party who makes a claim ... has the burden of proving it. This burden of proof means that the plaintiff must prove every element of [his/her] claim by a preponderance of the evidence.” This general instruction by itself is ambiguous, though based on our judgment call treating “every” to mean “every one separately,” it suggests a conjunction problem. But when instructing on individual causes of action, the District of Columbia makes the conjunction problem more explicit:

[Plaintiff] alleges that [Defendant] was negligent and is liable for [Plaintiff’s] harm. A negligence claim has three elements:

1. [Defendant] did not use ordinary care;
2. [Defendant’s] failure to use ordinary care caused [Plaintiff’s] harm; and
3. [Plaintiff] is entitled to damages as compensation for that harm.

[Plaintiff] must prove each element by a preponderance of the evidence—that each element is more likely so than not so. If [Plaintiff] proves each element, your verdict must be for [Plaintiff]. If [Plaintiff] does not prove each element, your verdict must be for [Defendant].

The italicized language treats such proof of the elements as a sufficient condition for the plaintiff to win: that is, meeting the condition obligates the jury to find for the plaintiff.

This does not exhaust the conjunction problem, however. Several jurisdictions—seven in civil and six in criminal instructions—direct juries to find for the claimant when the burden of proof has been met as to each separate element, without giving any whole claim instruction. In other words, they instruct on assumption 4b while omitting assumption 5. These jurisdictions—which we call “elements only, mandatory”—thus do not create an explicit conflict between elements-only and whole-claim decision rules. But we think they are best understood as replicating the conjunction problem for two reasons. First, one discussed aspect of the conjunction

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208. Id. § 5.01 (emphasis added).
problem is that jurors might well impose liability or guilt where the “elements only” condition has been met but the probability of the whole claim falls short of the burden of proof threshold. If that happens, that is indeed problematic. Second, it is widely believed that jurors will impose a whole claim condition (assumption 5) on themselves unless expressly instructed not to (and perhaps even then).

These mandatory “elements only” jurisdictions typically give general instructions that define the burden of proof as applying to “elements” or “propositions” rather than “claims,” such as this used in Illinois:

> When I say that a party has the burden of proof on any proposition, or use the expression “if you find,” or “if you decide,” I mean you must be persuaded, considering all the evidence in the case, that the proposition on which he has the burden of proof is more probably true than not true.209

The Illinois instructions then include this mandatory language after listing the substantive elements of a claim: “If you find from your consideration of all the evidence that each of these propositions has been proved, then your verdict should be for the plaintiff.”210 A handful of jurisdictions propound elements-only civil instructions differently, by incorporating element-by-element special verdict forms directly into their definitions of the burden of proof.211

In criminal cases, conjunction problem instructions are more common than in civil cases. Nine jurisdictions define their burden of proof in a manner suggesting that a conviction requires meeting the “beyond a reasonable doubt” standard for the charge as a whole, and then proceed to direct the jury—either in the general instruction or in the instruction on a specific substantive charge—to convict if “each element” has been proven. For example, Idaho instructs

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209. ILLINOIS PATTERN JURY INSTRUCTIONS—CIVIL § 21.01 (SUPREME COURT COMM. ON JURY INSTRUCTIONS IN CIVIL CASES 2017).
210. Id. § 21.02.
211. See TEXAS PATTERN JURY CHARGES—CIVIL § 200.3 (STATE BAR COMM. ON PATTERN JURY CHARGES 2016) (“After the closing arguments, you will go to the jury room to decide the case, answer the questions that are attached, and reach a verdict.... Answer ‘yes’ or ‘no’ to all questions unless you are told otherwise. A ‘yes’ answer must be based on a preponderance of the evidence.”); see also Schwartz & Sober, supra note 130, at 38-40 (showing similar practice by Minnesota, Mississippi, and Wisconsin).
generally that “the state must prove the alleged crime beyond a reasonable doubt.” But it then gives mandatory elements-only substantive instructions: “If you find that elements one(1)-five(5) above have been proven beyond a reasonable doubt, ... you must find the defendant guilty of first degree murder.” Pennsylvania replicates the conjunction problem in a single sentence by instructing generally that “it is the Commonwealth that always has the burden of proving each and every element of the crime charged and that the defendant is guilty of that crime beyond a reasonable doubt.” Six other jurisdictions follow the “elements only” pattern, typically defining the burden of proof as a requirement applying only to elements, and then requiring the jury to convict on proof of each element. For example, Virginia instructs generally that juries must “find the defendant not guilty unless and until the Commonwealth proves each and every element of the crime beyond a reasonable doubt”—which could be read as a permissive elements-only instruction. But after listing the substantive elements of specific crimes, Virginia instructs, “If you find from the evidence that the Commonwealth has proved beyond a reasonable doubt each of the above elements of the crime as charged, then you shall find the defendant guilty.”

3. Non-Conjunction-Problem Jurisdictions

The majority of jurisdictions use pattern jury instructions that avoid the conjunction problem by omitting assumption 4b. That is, they treat the elements as necessary but not sufficient conditions to find for the claimant, expressly or implicitly implementing the whole claim condition, assumption 5. A handful of jurisdictions treat claims holistically and do not purport to apply the burden of proof to separate elements. In all, thirty-six out of forty-seven

213. Id. No. 704A (emphasis added).
216. Id. No. G4.100.
jurisdictions use civil instructions, and thirty out of forty-five use criminal instructions, that avoid the conjunction problem.

Of the sixty-six sets of jury instructions that avoid the conjunction problem, most follow a pattern that is consistent with our “entailment check” interpretation. That is to say, they include an express or implicit “whole claim” condition, use language suggesting that the “each element” condition is necessary to find for the claimant, and omit language suggesting that the “each element” condition requires a finding for the claimant.

We have identified eighteen jurisdictions as using “entailment check” pattern in civil instructions and twenty-one in criminal instructions. This total of thirty-nine makes the entailment check the clear plurality pattern. Most entailment check instructions refer to the “claim,” “case,” or “charge” when defining the general burden of proof, implying that the burden of proof applies to the whole claim, and then go on to simply list the elements of specific substantive claims or offenses without suggesting that establishing the elements requires a finding for the plaintiff. For example, whole claim instructions might say “[p]laintiff ... has the burden of proving her case by a preponderance of the evidence,” or “[t]he party who asserts a claim has the burden of proving it by a fair preponderance of the evidence.” When coupled with instructions that list elements without directing a finding for the claimant, these are consistent with the entailment check. Some jury instructions simply state the claimant must prove “each,” or “each and every,” or sometimes “all” of “the following elements.” The Fifth Circuit, for example, introduces the substantive elements of a crime this way: “For you to find the defendant guilty of this crime, you must be convinced that

217. Pattern Jury Instructions (Civil Cases) § 3.2 (Comm. on Pattern Jury Instructions Dist. Judges Ass’n Fifth Circuit 2016) (emphasis added).

218. Connecticut Judicial Branch Civil Jury Instructions § 3.2-1 (Civil Jury Instruction Comm. 2008) (emphasis added); Indiana Model Civil Jury Instructions § 109 (Ind. Judges Ass’n 2016) (“Plaintiff must prove ... her ... claims by the greater weight of the evidence.”).

the government has proved each of the following beyond a reasonable doubt." The most natural reading of this language is that the finding based on the elements is permissive—that is, meeting the burden of proof on each element is necessary but not sufficient to find guilt. Entitlement check instructions typically go on to state that failure of the claimant to prove one element compels a finding for the defendant—a point that simply reiterates that proof of all the elements is a necessary condition for the claimant to win. Some do this as a general instruction, some do so with the specific claim or charge, and some do both.

The pattern we identify as “aggregate elements” consists of jury instructions that break out the elements but use language suggesting that the finding requires the elements to be viewed in the aggregate. Although express whole claim language is absent from these instructions, the aggregation language implies a whole claim condition. For example, the Eighth Circuit defines the burden of proof without reference to whole claims (“A fact has been proved ... if you find that it is more likely true than not true.”) and employs aggregation language to instruct on specific causes of action: “Your verdict must be for plaintiff ... and against defendant ... on plaintiff’s claim ... if all the following elements have been proved.” Other examples of this pattern merely list elements without stating whether the burden of proof applies to the elements individually or as a whole. For example, Alaska’s negligence instruction states:

In order to find that the plaintiff is entitled to recover, you must decide it is more likely true than not true that:
1. the defendant was negligent;
2. the plaintiff was harmed; and

220. PATTERN JURY INSTRUCTIONS (CRIMINAL CASES) § 2.19 (COMM. ON PATTERN JURY INSTRUCTIONS DIST. JUDGES ASS’N FIFTH CIRCUIT 2015).
221. See, e.g., ARKANSAS MODEL JURY INSTRUCTIONS—CIVIL § 203 (“If ... you find from the evidence that any of these propositions has not been proved, then your verdict should be for (defendant).”).
222. See, e.g., PATTERN JURY INSTRUCTIONS (CRIMINAL CASES) §§ 1.05, 2.19 (COMM. ON PATTERN JURY INSTRUCTIONS DIST. JUDGES ASS’N FIFTH CIRCUIT 2015).
224. Id. § 5.40.
3. the defendant’s negligence was a substantial factor in causing the plaintiff’s harm.\textsuperscript{225}

We agree with Nance’s argument that these instructions can be read as treating the “each element” condition as necessary but not sufficient, though they are admittedly ambiguous.\textsuperscript{226} Those instructions that direct a finding for the claimant when “all” the elements have been proved can be read as directing their mandatory language—the required finding—to the whole-claim condition.

The pattern we identify as “elements only—permissive” asks jurors to apply the burden of proof to separate elements, but does so without stating or implying that meeting the “each element” condition requires a finding for the claimant. For example, New Mexico instructs generally that the plaintiff “has the burden of proving every essential element of the claim ... by the greater weight of the evidence,”\textsuperscript{227} and simply lists the elements of specific substantive claims.\textsuperscript{228} At the same time, these instructions do not say anything to override a “whole claim” condition.\textsuperscript{229} This is important because story model research suggests that assumption 5—the claimant must prove the whole claim to the relevant standard of proof—is simply built into jurors’ reasoning, to the extent that only an express contrary instruction could override it.\textsuperscript{230} Eight sets of jury instructions—seven in civil cases and one in criminal cases—follow this pattern.

These two patterns, “aggregate elements” and “elements only—permissive,” do not merely avoid the conjunction problem. We believe they are consistent with the “entailment check” explanation, because such instructions taken as a whole leave in place the tacit understanding of a whole claim condition, while describing elements in terms that make them necessary but not sufficient conditions. Such instructions formally allow jurors to consider

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{225} \textit{Alaska Civil Pattern Jury Instructions} § 3.01 (Civil Pattern Jury Instructions Comm. 2017).
\item \textsuperscript{226} See Nance, supra note 18, at 950-51.
\item \textsuperscript{227} \textit{New Mexico Uniform Jury Instructions—Civil} § 13-304 (Unif. Jury Instructions—Civil Comm. 2017).
\item \textsuperscript{228} E.g., \textit{id.} § 13-1114.
\item \textsuperscript{229} See \textit{id.} §§ 13-304, 13-1114.
\item \textsuperscript{230} See Pennington & Hastie, supra note 7, at 527-28 (explaining that jurors probably consider whole claims whether instructed to or not).
\end{itemize}
\end{footnotesize}
individual elements for the purpose of ensuring that none fall below
the burden of proof, while basing their ultimate decision on the
probability of the whole claim. Nineteen sets of instructions (four-
teen civil, five criminal) fall into these two patterns.

The last pattern, which we have designated as “holistic,” com-
prises instructions that do not break claims down into elements to
be considered separately. Instead, the instructions identify causes
of action as a detailed definition rather than a list of elements. The
New York instruction on legal malpractice illustrates:

To establish CD’s liability for malpractice, AB must prove, by a
preponderance of the evidence, that, in representing AB in [state
matter in issue] CD failed to exercise that degree of care, skill
and diligence commonly exercised by a member of the legal pro-
fession .... In determining the degree of skill commonly used by
an ordinary member of the legal profession in CD's situation,
you should consider all of the evidence you have heard .... Once
you have determined the degree of skill commonly used by an
ordinary member of the legal profession in CD’s situation, you
should go on to decide whether CD departed from that standard .... AB has the burden to prove, by a preponderance of the evi-
dence, that CD did not, in fact, exercise that degree of skill, care
and diligence commonly used by an ordinary member of the legal
profession in the situation. 231

Holistic instructions can be understood to incorporate assumption
5 (the whole claim condition) while omitting assumption 4 (the each
element condition). Thus they cannot be explained as incorporating
the entailment check idea, since that assumes separate application
of the burden of proof to the elements individually. At the same
time, by focusing only on the whole claim condition, holistic instruc-
tions also avoid the conjunction problem. 232 Eight sets of jury in-
structions (four civil, four criminal, for a total of 8.7%) follow this
pattern.

To summarize: The data from the ninety-two sets of jury instruc-
tions (forty-seven civil, forty-five criminal) across fifty-five state and
federal jurisdictions show that only twenty-six (28.3%) replicate the

231. NEW YORK PATTERN JURY INSTRUCTIONS—Civil § 2:152 (COMM. ON PATTERN JURY
INSTRUCTIONS ASSN OF SUPREME COURT JUSTICES 2016).
232. See supra Part II.C.1 (explaining the holistic solution to the conjunction problem).
conjunction paradox. A total of fifty-eight (63%) are consistent with the entailment check explanation.

4. Special Verdict Forms

A general verdict is one that announces the guilt or liability result for each charge or claim: “on the claim of negligence, we find for the plaintiff” or “on the charge of murder in the first degree, we find the defendant guilty.”233 In contrast to the general verdict, a jury may be asked to render a “special verdict,” which breaks down the overall liability finding into a series of components.234 This is accomplished by requiring the jury to answer a series of questions on a written form, and the set of answers will dictate the ultimate verdict.235 Typically, the questions on a special verdict form will track the elements of the claim, though they might be presented as a series of case-specific factual propositions. For example:

1. Was [name of defendant] negligent?
   _ Yes _ No
   If your answer to question 1 is yes, then answer question 2. If you answered no, stop here, answer no further questions, and have the presiding juror sign and date this form.

2. Was [name of defendant]’s negligence a substantial factor in causing harm to [name of plaintiff]?
   _ Yes _ No
   If your answer to question 2 is yes, then answer question 3. If you answered no, stop here, answer no further questions, and have the presiding juror sign and date this form.

3. What are [name of plaintiff]’s damages?236

While many special verdict forms are longer because they embrace more than three elements,237 the above format is typical. In this

234. See FED. R. CIV. P. 49(a) (describing special verdicts).
235. See Special Verdict, BLACK’S LAW DICTIONARY, supra note 233.
236. JUDICIAL COUNCIL OF CALIFORNIA CIVIL JURY INSTRUCTIONS VF-400 (JUDICIAL COUNCIL OF CAL. ADVISORY COMM. ON CIVIL JURY INSTRUCTIONS 2016).
237. See, e.g., id. at VF-401 (listing six elements on a special verdict form).
format, the “no” answer indicates a failure to meet the burden of proof on a given element, which results in a defense verdict. A plaintiff’s verdict requires a series of “yes” answers.

As the above form suggests, the special verdict form seems to imply that meeting the burden of proof on each element seriatim is a sufficient condition to render a plaintiff’s verdict. On this type of form, jurors have no straightforward opportunity to answer “yes” to the liability questions yet still find for the defendant. (They could do so somewhat indirectly by entering zero money damages in some cases.) Depending on how the form is written, and how juries use the form to structure their inquiries, the special verdict form could create an additional layer of ambiguity. If jurors follow their instincts or instructions to decide the whole claim first, the special verdict form could simply be used to retrofit findings on the elements. But it is also conceivable that judges and jurors construe special verdict forms as imposing condition 4b, so that a finding on “each element” directs the jury to find for the claimant.

D. Reconsidering the Conjunction Problem: Theory Meets Practice

We are tempted to say that there really is no conjunction problem. On the surface, the problem is easily fixed by dropping assumption 4b: that is, by instructing juries that the overall claim must be proved to the probability threshold, and that proving each element to that threshold is a necessary condition, while refraining from stating that it is a sufficient condition. To the extent that the probability gap troubles theorists, we are not uncomfortable in dismissing that concern as a math or perception mistake. But that’s all in theory.

In practice, matters are more complicated. On the one hand, most jurisdictions give instructions that apply the burden of proof to whole claims, and either omit reference to elements or else refer

238. The Federal Rules imply that a general verdict form can ask a series of underlying factual questions and yet generate a general verdict that is “inconsistent” with those answers. See Fed. R. Civ. P. 49(b)(3). This illustrates that a verdict form can be written to embody both conditions 4a and 5.

239. See supra Table 1.

240. See, e.g., Connecticut Judicial Branch Civil Jury Instructions § 3.2-1 (Civil Jury Instruction Comm. 2008).
to them in ways consistent with the entailment check explanation.\textsuperscript{241} These jurisdictions, therefore, do not reproduce the conjunction problem. By our tally, this accounts for sixty-six out of the ninety-two sets of pattern jury instructions we analyzed (72%). On the other hand, the number of jurisdictions whose instructions reproduce the conjunction problem is not trivial. And the use of special verdict forms adds to this ambiguity, because they can be understood to mandate a win for the plaintiff based on the probability of elements rather than the whole claim. It is not certain how widespread the use of special verdicts actually is.\textsuperscript{242}

Moreover, we find it noteworthy that the conjunction problem is more prevalent in criminal instructions. The jurisdictions we studied are 42% more likely to require juries to convict in criminal cases based on meeting an “each element” condition (assumption 4b) than they are to impose the analogous “each element” requirement on civil juries.\textsuperscript{243} What to make of this is unclear. The multiplication rule for conjunctions—even assuming dependence of the elements—suggests, in theory, that “each element” instructions tend to favor the claimant.\textsuperscript{244} It might thus be the case that many jurisdictions are adding a thumb to the scale on the side of the prosecution in criminal cases. The question is certainly worth further study.

We hesitate to draw any real-world conclusions about how legal practice should be changed relative to probability issues. Yet two conclusions are very clear. First, many evidence theorists have approached the conjunction problem from the wrong direction: as a theoretical problem whose solution requires a major theoretical shift with potential implications for legal practice.\textsuperscript{245} As argued above,

\textsuperscript{241.} See, e.g., ARKANSAS MODEL JURY INSTRUCTIONS—CIVIL § 203 (ARK. SUPREME COURT COMM. ON JURY INSTRUCTIONS—CIVIL 2016).

\textsuperscript{242.} Special verdicts are virtually never used in criminal cases, and their use in civil cases is far from universal. See 9B WRIGHT & MILLER, supra note 37, § 2505 (noting that use of special verdict forms has not been established through empirical study and explaining that “[r]egardless of how advantageous the use of the special verdict may be, the use of Rule 49(a) never has been widespread in the federal courts”); Anne Bowen Poulin, The Jury: The Criminal Justice System’s Different Voice, 62 U. CIN. L. REV. 1377, 1420 (1994) (noting the rarity of special verdict forms in criminal cases).

\textsuperscript{243.} Numerically, fifteen out of forty-five jurisdictions implement assumption 4b in their criminal instructions (33.3%), compared to only eleven out of forty-seven jurisdictions doing so in their civil instructions (23.4%). See supra Table 1.

\textsuperscript{244.} See supra note 8 and accompanying text (explaining the multiplication rule).

\textsuperscript{245.} See supra note 15 and accompanying text.
the theoretical proposals to change or omit the rules of probability, or to drop the assumption that a claimant must prove his overall claim to the applicable probability threshold, could require significant changes in how we think about the litigation system and how that system currently operates.246 Attaining theoretical consistency can produce unintended and undesirable practical consequences.

Second, and related, the conjunction problem should be understood and studied from an empirical vantage point. Many evidence scholars, even those whom we think are overly concerned with the theoretical impact of the conjunction problem, seem to agree that jurors will tend to decide cases holistically, based on their level of belief in the claim as a whole, if not instructed otherwise.247 Indeed, many believe that juries will decide cases based on the “whole claim” condition no matter what the jury instructions or special verdict forms say. Jurors’ answers to questions about the degree of proof of specific elements may thus be simply backed out of the product. That is to say, as far as the legal system is concerned, if the jury finds civil liability 51% true, or criminal guilt 91% true, the probabilities of the elements can be retrofitted to make the math come out right.

If that is true, then the conjunction problem does not exist in practice, and we can satisfy ourselves (for better or worse) with our explanation in this Article that it does not create a significant problem in theory. Certainly it does not create a theoretical problem that provides a good reason for banning probability theory as an analytical tool in understanding adjudicative fact-finding.

But empirical studies might suggest that there is a conjunction problem after all. For example, prosecutors’ extremely high rate of convictions in criminal trials might not be the result of a desirable selection effect—that prosecutors use their discretion to try only the strongest cases.248 It could also result in part from a tendency of jurors to convict based on the probabilities of elements alone without considering the probability of the whole claim—or from the tendency in many jurisdictions to instruct jurors to do so. The fact

246. See supra Part II (describing the problems that may result from evidence theorists’ high-level theoretical approaches to solving the conjunction problem).
247. See supra Part II.C.1 (explaining some scholars’ theories that jurors decide cases holistically).
248. See supra note 30 and accompanying text.
that different instructions are used in different jurisdictions—some
directing a guilty verdict based on elements,249 others based on
whole claims,250—suggests the possibility for a natural empirical
experiment. Similar experiments could be constructed in civil cases.

It’s also possible that the effects of “each element” instructions on
juries may turn out to have empirical results that work well in prac-
tice despite seeming perverse from a purely probabilistic vantage
point. Viewed purely as a math problem, an instruction or special
verdict form indicating that the plaintiff wins if each element is
proven to the probability threshold should work to the advantage of
plaintiffs, if the probability of the whole claim is disregarded. But
suppose that empirical and policy research were to conclude that
juries too often reach defense verdicts in civil cases. After all, story
model research suggests that juries do not disregard whole claim
probabilities.251 In such a context, the use of a special verdict form
might offset a thumb on the scale of civil defendants. On the other
hand, plaintiffs’ lawyers frequently argue that complex, multi-
element verdict forms favor the defense, because they tend to
confuse jurors.252 If this is true, then probability theory and real-
world behavior would point toward contrary solutions.

The logic of our argument would suggest that a theoretical con-
junction problem could be cleared up by eliminating special verdict
forms and having all jurisdictions adopt a model in which juries are
instructed to decide whole claims and “check” for the probable
presence of each element. Yet this logic is hardly sufficient to suggest
actually making such a change to jury instructions without ade-
quate knowledge of what consequences would be brought about by
such a change. Perhaps the intuition is right, that plaintiffs have a
hard enough time proving each element without requiring proof of

249. See, e.g., NEW MEXICO UNIFORM JURY INSTRUCTIONS—CIVIL § 13-304 (Unif. Jury
Instructions—Civil Comm. 2017) (party seeking recovery must “prove[e] every essential
element of the claim”).

250. See, e.g., PATTERN JURY INSTRUCTIONS (CIVIL CASES) § 3.2 (Comm. on Pattern Jury
Instructions Dist. Judges Ass’n Fifth Circuit 2016) (“Plaintiff... has the burden of proving
[his/her/its] case”).

251. See Pennington & Hastie, supra note 7, at 527-28.

252. See Thornburg, supra note 27, at 1886-88 (citing research suggesting that special
verdict forms can work against plaintiffs). But see Spottswood, supra note 6, at 267 & n.36
(citing arguments from judges that special verdict forms offset a tendency of jurors to tilt
holistic decision-making against plaintiffs).
the whole claim—we simply don’t know. Theorists of any stripe should proceed with caution and modesty when it comes to extending theoretical insights to proposals for legal reform.253

CONCLUSION

The conjunction problem is discussed in evidence literature as a component of a larger debate about the role of probability theory in adjudicative fact-finding. We have argued that the conjunction problem is exaggerated, if not illusory, as a theoretical matter. It seems to be based in part on a mathematically wrong intuition: that it is somehow improper to require that elements be proven to a higher level of certainty than whole claims. This error is made worse by the erroneous application of the multiplication rule for independent events.254 And the conjunction problem is based formally on the legally questionable assumption that fact-finders are required to find for claimants who prove only that the elements of their claims exceed the applicable burden-of-proof probability threshold. In the language of most jury instructions, such proof of elements is a necessary but not sufficient condition to find for the claimant. If the applicable burden of proof applies to the overall claim, and jurors are not expressly directed to find for claimants who have proven each separate element, the jury instructions ask for an “entailment check.” That is, they can be understood to single out elements simply to make sure that those are included in the claimant’s more detailed factual narrative, and to remind the jury that failure of proof on one element entails rejection of the entire claim.

Our argument diminishing the conjunction problem removes a source of confusion from the larger theoretical debate about the role of probability theory in the adjudication system. Whatever grounds there are to “believe that adjudicative fact-finding is fundamentally incompatible with mathematical probability,”255 those grounds should not include the perceived need to “solve”—or even avoid—the conjunction problem. We might go a step further and suggest that

253. For this reason, we disagree with—or at least, we find premature—Spottswood’s conclusion that jury instructions should be changed to conform to probability theory. See Spottswood, supra note 6, at 292-96.
254. See supra Part III.
255. Allen & Stein, supra note 1, at 562.
the existence of paradoxes does not necessarily entail the total rejection of probability theory. For example, the “Blue Bus”\textsuperscript{256} and “Gatecrasher”\textsuperscript{257} paradoxes both tell us, not that probability theory is anomalous, but that naked statistical evidence—and perhaps purely probabilistic analysis—may be an incomplete basis for understanding how the concept of justified belief should work in the litigation system. There is a difference between using probability theory as an illustrative or analytical tool and using probability theory as a sufficient basis for determining what the fact-finder should believe.

\textsuperscript{256} See supra note 25 (discussing the “Blue Bus” paradox).

\textsuperscript{257} See Allen & Stein, supra note 1, at 573-74 (summarizing the “Gatecrasher” paradox).