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Extending the Taxation-of-Risk Model to Timing Options and Marked-to-Market Taxes

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APPENDIX

I. INTRODUCTION

In form, the income tax applies to economic outcomes, not prospects. It purports to distinguish economic winners and losers.[1] If I buy an ounce of gold today and sell in one year, my tax bill depends on how much I gain or lose. My initial investment in gold is relevant but only as a baseline, used to measure my ultimate gain or loss.

In substance, though, income taxes may not weigh on outcomes at all. The "taxation-of-risk" model challenges the idea that the income tax can effectively distinguish winners and losers,[2] claiming that taxing income (i.e., outcomes) is no different from taxing starting wealth (i.e., prospects). First developed in the 1940s to describe investor behavior,[3] the model has

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[1] *Cf.* Michael J. Graetz, *Implementing a Progressive Consumption Tax*, 92 HARV. L. REV. 1575, 1601 (1979) ("[L]ucky gamblers are not the same as unlucky gamblers"); *see also* David A. Weisbach, *The (Non)Taxation of Risk*, 58 TAX L. REV. 1, 27 (2004) (quoting Graetz in the context of taxation of risk).

[2] *See* Louis Kaplow, *Taxation and Risk Taking: A General Equilibrium Perspective*, 47 NAT'L TAX J. 789 (1994); Weisbach, *supra* note 1.

[3] Evsey D. Domar & Richard A. Musgrave, *Proportional Income Taxation and Risk-Taking*, 58 Q. J. ECON. 388 (1944).

been deployed recently to support proposals for fundamental[4] and incremental tax reform.[5] The income tax certainly burdens a winning investment with a higher tax bill. But, it aids a losing investment with a loss deduction, which potentially reduces taxes on other activities. By aiding losers and burdening winners, the income tax shifts some investment risk from investors to the government and makes a risky investment (e.g., an ounce of gold) less risky.

Investors can reverse this risk sharing by taking on more investment risk. In other words, they can "opt out" of taxes. An investor who would have bought a single ounce of gold absent the income tax would buy more. To fund the additional purchase, the investor might borrow (and pay interest) or liquidate low-risk assets (and forgo interest). Either way, the cost of opting out comes from interest applied to starting wealth. Parts II, III, and IV develop this result, which I will call the "classic model." The main finding is that the cost to opt out is fixed and based on starting wealth, not on actual gains or losses. Under the classic model, the investor would be indifferent between a wealth tax and an income tax.

The classic model, though, looks only at individual taxpayers. It does not press the important policy issue: is the *government* indifferent between wealth and income taxes? Part V suggests that the answer is "no." To opt out of the income tax, an investor must increase her investment risk. She could buy more of the investment (e.g., more gold), or she could increase her exposure to the investment with a derivatives contract (e.g., forward contract on gold). Either way, she must find a counterparty to supply the additional risk, and the counterparty has her own tax consequences. Part V shows that a wealth tax and an income tax produce the same public revenue only in one narrow and unlikely circumstance—when the government itself acts as counterparty to the investor who ops out. Private investors, acting without government cooperation, may affect their own private returns. They will not, however, transform overall income-tax revenue into wealth-tax revenue.

If the classic model does not address the ultimate goal of taxation— raising money for the fisc—then what does it do? Part VI shows that the classic model allows us to estimate the burdens of taxation as felt by

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[4] *See* Joseph Bankman & Thomas Griffith, *Is the Debate Between an Income Tax and a Consumption Tax a Debate About Risk? Does It Matter?* 47 TAX L. REV. 377 (1992); Noël B. Cunningham, *The Taxation of Capital Income and the Choice of Tax Base*, 52 TAX L. REV. 17 (1996); Deborah H. Schenk, *Saving the Income Tax with a Wealth Tax*, 53 TAX L. REV. 423 (2000); Alvin C. Warren, Jr., *How Much Capital Income Taxed Under an Income Tax Is Exempt Under a Cash Flow Tax?*, 52 TAX L. REV. 1 (1996).

[5] *See, e.g.*, Ethan Yale, *Investment Risk and The Tax Benefit of Deferred Compensation*, 62 Tax L. Rev. 377 (2009).

individual taxpayers ex ante. This result, in turn, lets us estimate just how much distortion an income tax has on investment decisions (at least when compared with a wealth tax). In other words, the classic model is perfectly plausible as a model of private behavior even if it breaks down as a model of public revenue. According to the classic model, the income tax does not impose higher or lower burdens on investments based on risk or expected return. As a normative matter, this result is highly desirable as we should hope that taxes do not interfere with economic decisions, like investing in high or low risk assets.

Part VII shows, however, that this happy result fails as we begin to account for strategic behavior by investors. The classic model assumes that investors receive a significant tax benefit for losses but do not behave strategically by timing their gains and losses. If investors can accelerate (or "harvest") their losses and defer gain, they hold a "timing option" with respect to taxes. With higher risk, losses (and gains) are more likely. Thus, timing options give investors a reason to prefer risky assets.

Policymakers understand that timing options threaten the fisc, and they respond by limiting loss deductions. In general, investors in the United States must match their capital losses against their capital gains. If they have no capital gains, they must defer their capital losses until they can match losses against gains. If they never have gains, they never deduct their losses. Part VIII shows how loss limitations not only counter timing options but create a new problem. They cause investors to disfavor risky assets because losses may not be fully usable.

Dissatisfied by the deferral opportunities of realization-based taxes and the distortions just described, many commentators would prefer to tax gains and losses on an annual basis, regardless of whether investors "realize" gain or loss with a sale. Part IX shows that the taxation-of-risk model extends to accretion taxes as well. Because taxpayers can opt out of accretion taxes, they may not be inherently fairer or better reflective of income than realization taxes. Both reduce to a charge that the investor may prepay.

II. THE TAXATION-OF-RISK MODEL HAS DEFENSIBLE ASSUMPTIONS

A. Introduction

Ultimately, this article will interpret the taxation-of-risk model as identifying the private cost of opting out of taxation. We first identify the nature of the tax system under consideration. Doing so will involve assumptions, some of which are "unrealistic" in that they do not reflect an actual tax system. The Internal Revenue Code, for example, limits the

ability of taxpayers to deduct capital losses[6] while the taxation-of-risk model assumes that losses and gains receive symmetrical treatment.[7] This article will have much more to say about losses below and indeed develops a more complex model that accounts for loss limitations.[8]

Even without the more complex models, the simple model supplies theoretical value that describes the basic structure of taxes. At the heart of the actual income tax, with all of its political ugliness and administrative concessions, lie a few theoretical ideals. Taxpayers should pay tax on the amount of gain they have. True economic losses should reduce tax liability. This part seeks to model these lofty ideals, unobscured by political compromise or administrative necessity.

B. The Path of an Asset Value in a World Without Tax

We will model the tax consequences to an investor of holding an asset (or entering into a derivatives contract) over some period. Let us specify that the period lasts T years, running from time $= 0$ to time $= T$. The initial value of the asset is S_0. When the investor sells, it is worth S_T. This article uses an arrow to show how an investor's wealth changes over time. In a tax-free world, we simply say

$$(1) \qquad\qquad\qquad S_0 \rightarrow S_T$$

Note that we do not actually need to specify much about S_T, other than to note that it is "random." The asset could be anything: an ounce of gold, a share of Facebook stock, etc. S_0 is the value today (time $= 0$), and S_T is the value at some time in the future (time $= T$). When describing assets, we usually think of both S_0 and S_T as being positive. A huge swath of investment activity, however, does not follow this restriction. Derivatives contracts often involve no up-front investment ($S_0 = 0$) but can have a future value, S_T, that is either negative or positive.[9]

C. A Simple Income Tax and Model

1. Investors Have a Fixed Holding Period

For now, we will assume that investor sells the asset at time $= T$. Ex

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[6] *See* I.R.C. § 1211.

[7] *See infra* part II.C.4.

[8] *See infra* part VIII.

[9] *See infra* part V.B.

ante, the value of the asset upon sale, S_T, is not known. The holding period until sale (T itself) is known. For example, a taxpayer may buy an ounce of gold planning to hold it for ten years, but the investor has no way of knowing the price of gold in ten years.

Assuming a fixed holding period may seem rigid, and we will relax it later. The assumption is useful, though, for two reasons. Specifying the holding period up front allows us to determine the cost of taxes up front. Moreover, a fixed holding period precludes strategic behavior known as "timing options" that allows taxpayers to accelerate losses and defer gains.

2. Investors Face Tax Consequences Only at the End of This Holding
   Period

All tax consequences arise at time $= T$ when the investor sells. This "realization" method of taxation is the worldwide standard for taxing the increase or decrease in the value of investment assets.[10] Investors do, however, pay tax on dividends and interest when received.[11] For now, we will assume that the asset pays no dividends, interest, or the like.[12]

3. The Government Imposes Tax at a Flat Marginal Rate

In examples, this article assumes that the government imposes a flat tax of twenty or fifty percent. When formalizing results, it expresses the rate as τ. The taxation-of-risk model remains viable when extended to varying rates of taxation (e.g., on different asset classes)[13] and to progressive taxation.[14]

4. The Government Treats Gains and Losses Symmetrically

Under a fifty percent tax rate, $100 of gain produces a $50 tax. Under the taxation-of-risk model, a $100 loss produces a $50 tax benefit. The government shares in both the return and the risks of private investment. The model assumes that gains and losses receive symmetrical treatment. U.S. tax laws are not, however, symmetrical, because taxpayers face limits

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[10] *See* HUGH J. AULT & BRIAN J. ARNOLD, COMPARATIVE INCOME TAXATION 232 (3d ed. 2010).

[11] *C.f., e.g.*, I.R.C. § 61(a)(4)–(6) (including interest, rents, royalties, and dividends in gross income).

[12] *But cf. infra* part IX (extending the model to annual accretion taxation).

[13] *See* David A. Weisbach, *Taxation and Risk-Taking with Multiple Tax Rates*, 57 NAT'L TAX J. 229 (2004).

[14] *See* Lawrence Zelenak, *The Sometimes-Taxation of the Returns to Risk-Bearing Under a Progressive Income Tax*, 59 SMU L. REV. 879 (2006).

on their ability to deduct losses.[15] Arguably, these limits are administrative requirements, imposed in order to control strategic behavior.[16] But for the presence of strategic behavior, taxpayers should receive loss deductions. As we initially preclude strategic behavior, we can also assume that losses receive symmetrical treatment.

We can now describe the path that starting wealth takes under the simple tax. As in a world without tax, our investor starts with an asset worth S_0[17] and holds an asset worth S_T at time = T. At that time, the investor sells and faces a flat tax rate of τ (a placeholder for 20%, 50%, etc.). If there is a gain, the investor pays tax of $(S_T - S_0)\tau$ If there is a loss, the investor receives a symmetrical tax benefit of $(S_0 - S_T)\tau$ We could equivalently say that the investor always "pays" the government $(S_T - S_0)$, but that this "payment" is negative if there is a loss.

After taxes, our investor's starting wealth of S_0 faces a different "path" than the one in a tax-free world:

$$S_0 \rightarrow S_T - (S_T - S_0)\tau = S_T(1 - \tau) + S_0\tau$$

(2)

This expression shows that the investor no longer enjoys full ownership of the asset after tax. In particular, note that ending wealth is less sensitive to S_T, the value of the asset at the end of the holding period. In a tax-free world, the investor's ending wealth is S_T. In a taxable world, the correlation is diminished to only a portion of asset's final value.

5. The Cost of Funding Additional Asset Purchases Is Measured byInterest Rates

The model will later show that investors can preemptively offset (or "opt out" of) tax by increasing their exposure to the risky investment asset considered above. Of course, the investor must find funding for the additional exposure to the asset. Suppose that the risky asset considered above is gold. How does the investor get more gold? The investor could shift funds out of cash holdings (like money market funds and certificates of deposit) and into the risky investment asset. Low-risk assets typically bear interest, which the investor would forego in order to buy the risky investment asset. Or, the investor could borrow additional funds, paying whatever interest rate the market demands. The two methods are equivalent so long as the rates to borrow and to lend are equivalent.

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[15] *See infra* part VIII.A. (discussing reasons for loss limitations).

[16] *See id.*

[17] *Cf. supra* part II.B. (describing a tax-free world).

If rates are not equivalent, we would expect investors to pursue the cheaper route. An investor holding cash yielding 2% a year should not borrow at 8% a year to fund additional asset purchases. She should simply liquidate the 2%-interest-bearing asset to buy more of the risky asset. The "cost" of buying the risky asset is foregone interest of 2% a year.[18] Even if the investor never actually owned the low-risk 2% asset, it nevertheless represents the opportunity cost of the risky investment, the assured rate of interest the investor could otherwise receive. Current rates are historically low. Short-term U.S. Treasury securities yield almost no interest;[19] LIBOR (a rate financial institutions pay each other) ranges from next to nothing for daily rates to about one percent for yearly rates.[20]

This article will follow two simplifying conventions when describing the cost of buying more of the risky asset. First, we will usually say that the investor "borrows" funds to make the purchase. We could equivalently say that the investor liquidates existing low-risk assets, but the idea of borrowing is more direct and vivid. Both approaches are equivalent so long as we make the simplifying assumption that the interest rates for borrowing and investing are the same. Second, we will specify the rates as being after tax and assume that interest received is taxable and that interest paid is deductible.[21] While published rates are almost always subject to tax, the true measure of the cost of buying a risky asset is what the investor would have received, after tax, by investing in cash. Examples use an after-tax rate of 2%. In models, this article will use an after-tax rate of r.[22]

6.   Taxes Do Not Affect Asset Prices

We will disregard the effects of taxes on asset prices. In other words,

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[18]   For large financial institutions, the rates for borrowing and lending are roughly the same. Usually they may borrow or lend at something like the London Interbank Offered Rate (LIBOR), a very low rate comparable to the risk-free rate on U.S. Treasury securities. *See* JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 77 (8th ed. 2012).

[19]   In April 2012, one month Treasury notes yield 0.05%, and thirty-year Treasury bonds yield 3.35%. *See Daily Treasury Yield Curve Rates*, U.S. DEP'T OF TREASURY, http://www.treasury.gov/resource-center/data-chart-center/interest-rates/
Pages/TextView.aspx?data=yield (last visited Feb. 12, 2013).

[20]   *SeeLIBOR,   Other   Interest   Rate   Indexes*,   BANKRATE.COM,   http://www.
bankrate.com/rates/interest-rates/libor.aspx (last updated Feb. 13, 2013).

[21]   *Cf.* I.R.C. § 61(a)(4) (including interest in gross income) and I.R.C. § 163(a) (allowing a deduction for interest paid). This is similar to the assumption about the deductibility and includability of gains and losses. *See supra* part II.C.4.

[22]   Under a tax rate of τ, the before tax interest rate would be $r / (1 - \tau)$. Under a 50% tax, a pre-tax rate of 4% implies an after-tax rate of 2%. Under a 20% tax, a pre-tax rate of 2.5% implies an after-tax rate of 2%.

the asset price goes from S_0 to S_T regardless of taxation. This assumption helps us compare results for an investor operating in a taxable and a tax-free world. We will revisit this assumption later.[23]

III. A SIMPLE TAX EXEMPTS SIDE BETS (INCLUDING MANY DERIVATIVES CONTRACTS)

In this article, a "side bet" is some contract that initially has no value but can take either a positive or a negative value in the future. A wager is a simple example. An optimist, Ophelia, and a pessimist, Pete, decide to wager on the price of gold in one year. Ophelia thinks it will go up, and Pete thinks it will go down. They agree that the loser will pay the winner $100. In a tax-free world, the wager involves only Ophelia and Pete.

Taxes introduce a third party, the government, into the wager. Suppose the government imposes a 50% tax. The loser still pays the winner $100. But now, the winner pays a $50 tax to the government, and the loser receives a tax benefit worth $50. In this case, the government's tax claim adds nothing to the public revenue. The government receives $50 from the winner but pays $50 to the loser.

Loser
Pays Winner $100
Receives from Government $50
Nets –$50

Government
Receives from Winner $50
Pays to Loser $50
Nets -$0-

Winner
Receives from Loser $100
Pays to Government $50
Nets +$50
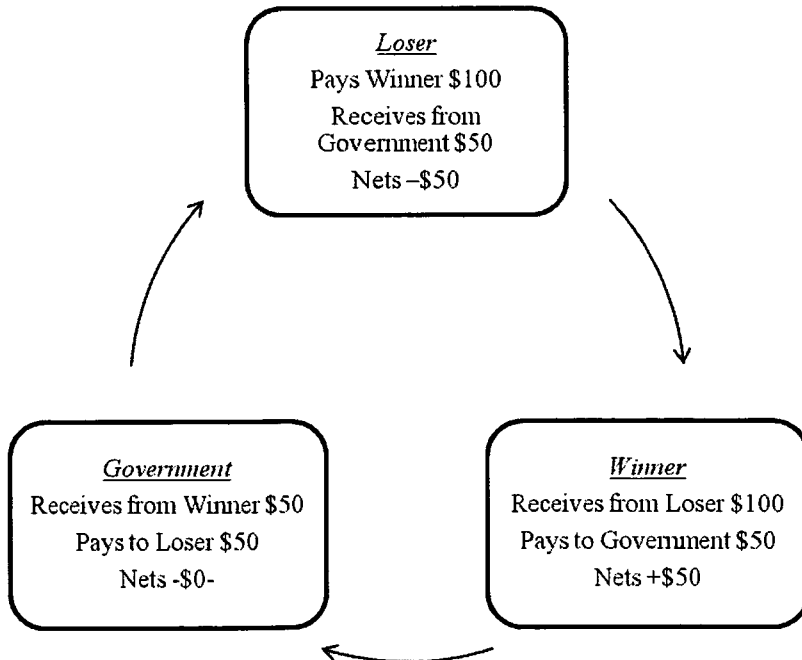
The tax gives the government nothing and merely alters Pete and Ophelia's deal. Pete and Ophelia can easily undo even these effects. In a

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[23] *See infra* part V.C; *see also* Kaplow, *supra* note 2.

tax-free world, they agreed to a $100 transfer. In 50%-tax world, they achieve the same result by doubling their bet. The winner receives $200 from the loser, but pays half to the government. The loser pays $200 to the winner, but receives a tax benefit of $100 from the government. As a result, we can say that Pete, Ophelia, and the government are all indifferent to the tax on the gold wager.
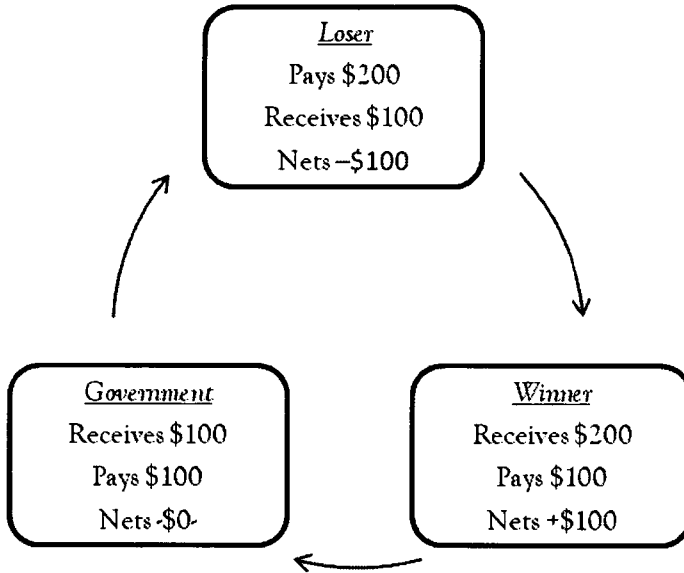
Let us now generalize this result. Parties facing a tax of τ can opt out of the tax by increasing the amount covered by the contract up by a percentage amount of $\tau / (1 - \tau)$. We have seen the results for a 50% tax applied to $100 desired bet already. Under a 20% tax and $100 desired bet, Pete and Ophelia increase their bet by $0.20 / (1 - 0.20)$ or 25%. The loser pays the winner $125. The winner pays the government 20% or $25. The loser receives a tax benefit of $25. The final result mirrors the tax-free world. The winner gets $100, the loser loses $100, and the government gets nothing. This general gross up factor, of $\tau / (1 - \tau)$, is the back-bone for more sophisticated opt-out transactions discussed throughout this article.

The result may have practical implications for the taxation of derivatives.[24] Many forms of derivatives contracts—including futures, forwards, and swaps—are side bets that have no initial value to either party[25] much like Ophelia and Pete's wager on the price of gold. Indeed, we

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[24] *See generally* David M. Schizer, *Balance in the Taxation of Financial Instruments: An Agenda for Reform*, 104 COLUM. L. REV. 1886 (2004).

[25] Options, though, involve the payment of a premium, making it more like an asset than a side bet.

could readily characterize their wager as itself a derivatives contract.[26] The market for derivatives contracts is staggeringly large. The value of world-wide derivatives contracts, excluding those traded on exchanges, is estimated at over $21 trillion,[27] 50% larger than either the entire U.S. economy or its public debt.[28] The proper tax treatment of derivatives has baffled policymakers and scholars for at least two decades.[29]The analysis of Pete and Ophelia suggests, however, a conceivable solution of exempting certain derivatives contracts from taxation altogether.

IV. INVESTORS CAN OPT OUT OF THE SIMPLE TAX ON INVESTMENTS BY PAYING A FIXED CHARGE

A. Opting Out

1. Achieving the Results froma Tax-Free World

The prior part dealt with side bets, and we saw that investors can readily opt out of taxation without cost. We now extend this result to actual investment assets. The basic concept remains the same. As with the side bet, investors must increase their exposure to the risky investment. The only difference is that, now, the investor must pay some interest charge to achieve the desired result.

We should make a few more formal notes about the desired result. In equation (1), we described how an asset would move in value in a tax-free world. An investor starts with S_0 and ends with S_T. In equation (2), we saw how a simple tax affects the investor. The government shares in the investor's gain or loss. This part describes how an investor might "opt out" of the tax system as described in part II.C. By opting out, I simply mean

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[26] Ophelia has written a "cash-or-nothing put" on gold. If gold drops below its current price, she pays $100. Pete has written a "cash-or-nothing call" on gold. If gold rises above its current price, he pays $100. The exchange of contracts between Ophelia and Pete would be financially "fair" if interest rates are zero. *See* HULL, *supra* note 18, at 581–82.

[27] *See Amounts Outstanding of Over-the-Counter (OTC) Derivatives*, BANK OF INTERNATIONAL SETTLEMENTS (Dec. 2012), http://www.bis.org/statistics/otcder/ dt1920a.pdf.

[28] *See The Debt to the Penny and Who Holds It*, TREASURY DIRECT, http://www.treasurydirect.gov/NP/BPDLogin?application=np (last visited Feb. 13, 2013). Similarly, the U.S. gross domestic product for 2010 was $14.76 trillion. *See National Income and Product Accounts Tables: Table 1.1.5 Gross Domestic Product*, BUREAU OF ECON. ANALYSIS, http://www.bea.gov/iTable/iTable.cfm?ReqID= 9&step=1 (last revised Jan. 30, 2013).

[29] *See* STAFF OF J. COMM. ON TAXATION, 112TH CONG., JCX-56-11, PRESENT LAW AND ISSUES RELATED TO THE TAXATION OF FINANCIAL INSTRUMENTS AND PRODUCTS1 (Comm. Print 2011).

that the investor wants to offset the risk-sharing inherent in income taxation. To the extent possible, the investor will try to bear the full gains and losses inherent in the asset S_0.

2. Opting Out with Zero Interest Rates

To highlight the role of interest rates in opting out, we initially suppose that interest rates are zero. Suppose that Ophelia owns an ounce of actual gold, worth $1000.[30] She plans on holding the gold for one year before selling. At the time of sale, she faces a simple 50% tax, under the assumptions described earlier. She will keep only half of her gains, but will bear only half of her losses. We focus on two scenarios for now. Gold might rise or fall by 20%.[31] Absent any adjustment, Ophelia enjoys the following returns after tax:

- If gold rises to $1200 per ounce, she keeps only $100 of her $200 gain. She has $1100 after tax.
- If gold falls to $800, she bears only $100 of her $200 loss. She has $900 after tax.

In effect, the government becomes a mandatory partner, sharing in half of all gains and losses[32] of Ophelia's gold. The government's interest does not, however, extend to Ophelia's initial capital of $1000.

Ophelia wants to nullify the tax, and the classic taxation-of-risk model gives her a way to do so. As in the prior subsection, she doubles her investment. She simply borrows $1000 for a year and buys another ounce of gold. For now, assume that she need not pay any interest.

- Gold rises to $1200 per ounce. Ophelia's two ounces are worth $2400. She pays a $200 tax on her $400 of gain, and repays her lender $1000. She is left with $1200.
- Gold falls to $800 per ounce. Ophelia's two ounces are worth $1600. She takes a deduction worth $200 because of her $400 loss. After she repays her lender, she is left with $800.

Whether gold goes up or down, Ophelia has an after-tax payoff equal to the value of one ounce of gold, her result in a tax-free world. She has effectively opted out of the tax.

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[30] This amount is used for ease of presentation.

[31] In rough terms, we can think of this as representing a standard deviation of twenty percent. This is roughly the current market volatility for gold. *See CBOE Delayed Market Quotes*, CHICAGO BOARD OPTIONS EXCHANGE, http://www.cboe.com/DelayedQuote/SimpleQuote.aspx?ticker=GVZ (last visited Feb. 11, 2013).

[32] *See* MYRON S. SCHOLES ET AL., TAXES AND BUSINESS STRATEGY 3 (3d ed. 2009).

3.   Opting Out with Positive Interest Rates

Our results are only slightly different with positive interest rates. Suppose that the prevailing interest rate is 2% (after tax).[33] To borrow the $1000 to finance her additional gold purchase, she would need to pay after-tax interest of $20. Or, she could liquidate $1000 of her risk-free assets, foregoing the certainty of $20 worth of after-tax interest. In either case, Ophelia must pay $20, or 2% of her additional investment, to opt out of the tax system. Beyond this fixed interest payment, her results are precisely the same as in the prior subsection. The opt-out transaction works no matter how risky gold is or where gold prices go next year. In one year, Ophelia will have wealth equal to the value of one ounce of gold (her result absent taxes) less $20. Ophelia simply pays (or forgoes) interest on $1000 (the initial value of an ounce of gold).

B. Retrospective Cost of Opting Out

We now formalize the result from the prior section. Doing so is not just an exercise in algebra. It allows us to extend the simple opt-out transaction to more complex tax regimes.

Recall that, if the investor does not opt out, the tax causes her initial asset to follow the path specified in equation (2).

$$(3) \qquad\qquad S_0 \rightarrow S_T(1 - \tau) + S_0\tau$$

Note that, on the right side of the path, the investor does not bear the full risk and rewards of S_T. She has only a portion, given by $(1 - \tau)$.

1.   Grossing Up Initially

To be effective, the opt-out transaction must result in the investor's receiving every dollar of gain and bearing every dollar of loss from changes in the value of the asset. To achieve this result, our investor needs more of the risky asset. If the investor starts with a "grossed up" amount of $S_0/(1 - \tau)$ rather than merely S_0, she will have the "right" amount of S_T at the end (time = T).

$$(4) \qquad\qquad \frac{S_0}{1 - \tau} \rightarrow \frac{S_T(1 - \tau) + S_0\tau}{1 - \tau} = S_T + \frac{\tau}{1 - \tau}S_0$$

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[33] *See supra* Part II.C.5.

Our investor, however, starts with only S_0. She needs extra funds to hold the grossed up amount $S_0 / (1 - \tau)$. The additional amount she needs is

(5)
$$\frac{S_0}{1-\tau} - S_0 = \frac{\tau}{1-\tau} S_0$$

For example, if τ is 20% and S_0 is $1000, the investor would need $1250 of the asset. She must increase her holdings by $250. She can obtain these extra funds by borrowing or by liquidating low risk assets (like a savings account or money market fund).

2. Repaying Later

The investor borrows the additional amount described in (5). At the end of the holding period, the investor must repay the debt. Using continuous compounding at an after-tax rate of r, the required repayment is—

(6)
$$\frac{\tau}{1-\tau} S_0 e^{rT}$$

For example, borrowing $250 at 2% for 10 years implies a repayment of $305.35. Similarly, if the investor liquidated a low-risk asset yielding 2%, she has forgone owning an asset that would be worth $305.35 in 10 years.

The investor's final (time = T) wealth will be the value of the grossed-up asset after tax (given by the left side of the arrow in (4)) less the amount of debt repayment (given by (6)).

(7)
$$S_T + \frac{\tau}{1-\tau} S_0 - \frac{\tau}{1-\tau} S_0 e^{rT} =$$
$$S_T - \frac{S_0(e^{rT} - 1)\tau}{1-\tau}$$

3. Final Results

Based on this result, we can say that the opt-out transaction leads to the following path for an asset to follow after tax:

(8)
$$S_0 \rightarrow S_T - \frac{S_0(e^{rT} - 1)\tau}{1-\tau}$$

We can compare the result in (1)(8) with the no-tax world described in (1). In (1), the investor is subject to no tax and has ending wealth of S_T. In

(1)(8), our investor faces a tax of τ but privately "opts out" of the tax. Her ending wealth is S_T less the following amount—

(9) $$\frac{S_0(e^{rT} - 1)\tau}{1 - \tau}$$

We can describe the amount in (1)(9) as the retrospective cost of opting out of taxes, the fixed cost of avoiding the impact of taxes, payable at time $= T$. What makes this retrospective cost interesting is that it does not depend at all on the final value of the asset, S_T. The cost depends on the initial value, the tax rate, interest rates, and the holding period. This result is the core of the classical "taxation of risk" model.

4. Example

Suppose our investor starts with an asset worth $1000. She faces a 20% tax, 2% after-tax cost of borrowing, and a 10-year holding period. Her retrospective cost should be $55.35. To see that this is correct, suppose that the asset could go up or down by 50% over the 10 years. In a tax-free world, she would have either $1500 or $500 at the end of 10 years. By opting out of tax, she would have the same amounts less the $55.35 retrospective cost. Depending on the performance of the asset, she will have either $1444.65 or $444.65.

- The investor grosses up to $1250, per (4). This amount will rise or fall by 50%.
- To obtain this amount, she borrows $250 (or liquidates low-risk investments).At the end of 10 years, she repays debt of $305.35, regardless of whether the asset goes up or down.
- If the grossed-up asset goes up by 50%, it is worth $1875. She has gain of $625 and pays tax of 20% or $125. The asset ($1875) less the tax bill ($125) less the debt repayment ($3050.35) leaves her with $1444.65, just as we predicted.
- If the grossed asset goes down by 50%, it is worth $625. She has a loss of the same amount and receives a tax benefit worth 20% of her loss or $125. The asset ($625) plus the tax benefit ($125) less the debt repayment ($305.35) leaves her with $444.65, just as we predicted.

C. Prospective Cost of Opting Out

The prior subsection described opting out in terms of paying a charge at the end of the holding period. This description best captures the idea of opting out of a realization tax because it shows the investor may opt out

indefinitely. The investor grosses up her holdings until she decides to sell in the future. Ultimately, she might sell in one year, ten years, or whenever. The cost of the opt-out transaction grows over time, but that cost will depend only on the starting wealth, the holding period, and the interest rate. The cost does not depend on the ending value of the asset.

If the investor knows her holding period up front, she can prospectively determine the cost of opting out. Above, (9) gave the retrospective cost (incurred at time = T) of opting out. The investor could conceptually "prepay" this amount at time = 0. Using continuous compounding, we discount the retrospective cost to present value by multiplying by e^{-rT}. Thus, the prospective cost is—

$$(10) \qquad S_0(e^{rT} - 1)\frac{\tau}{1 - \tau}e^{-rT} = S_0(1 - e^{-rT})\frac{\tau}{1 - \tau}$$

To see that this works, return to the example just given. Our investor starts with an asset worth $1000. She faces a 20% tax, 2% after-tax interest, and a 10-year holding period. Previously, we found her retrospective cost of opting out would be $55.35. The prospective cost, incurred at time = 0, is $45.32. To see why, again suppose that the asset could go up or down by 50% over the ten years. In a tax-free world, an initial investment of $1000 would be either $1500 or $500 at the end of ten years. By prospectively opting out of tax, she starts with an initial investment of $1000 plus the $45.32 cost, or $1045.32. She ends with an investment equal to her results in a tax-free world ($1500 or $500).

- The investor grosses up from $1045.32 to $1250. This amount will rise or fall by 50%.
- To obtain this amount, she borrows $204.68 (or liquidates low-risk investments). At the end of 10 years, she repays debt of$250, regardless of whether the asset goes up or down.
- If the grossed-up asset goes up by 50%, it is worth $1875. She has gain of $625 and pays tax of 20% or $125. The asset ($1875) less the tax bill ($125) less the debt repayment ($250) leaves her with $1500, the same as a tax-free world.
- If the grossed asset goes down by 50%, it is worth $625. She has a loss of the same amount and receives a tax benefit worth 20% of her loss or $125. The asset ($625) plus the tax benefit ($125) less the debt repayment ($305.35) leaves her with $500, the same as a tax-free world.

D. Incentives to Opt Out

Up to now, the model has described the *ability* to opt out of taxes. The

simple tax described above[34] mandates risk sharing between an investor like Ophelia and the government. Lower risk, though, will lower Ophelia's expected return, something she may not want. In responses, Ophelia may simply take on more risk through leverage or reallocating her portfolio.

Do investors actually respond to taxes in this way? The original work on the taxation of risk mainly concerned itself with investor behavior.[35] It may be impossible to know empirically. A randomized experiment can hardly compare tax-free and taxable worlds, and the model developed so far is quite idealized. It does not reflect the administrative concessions and ugliness found in real tax systems. The question of whether taxpayers would actually opt out asks us to predict and compare behavior in two purely hypothetical worlds (a tax-free world and one with a simple tax).

The simple model does, though, make it seem fairly straightforward and cheap for an investor to undo the risk-sharing that comes from the simple tax. Even if average investors do not explicitly understand the model, they might adapt their behaviors over time to the risk-sharing inherent in the simple tax. More importantly, the ability to opt out may tell us much about how taxes affect investor behavior. The "classic" taxation of risk model developed so far shows that the cost of opting out does not depend on the risk or expected return of assets. In this sense, the income tax does not discriminate against risky investments.

V. EVEN UNDER THE SIMPLE TAX, THE GOVERNMENT SHARES IN GAINS AND LOSSES FROM RISKY INVESTMENTS UNLESS IT OPTS OUT

A. Equivalence Between a Wealth Tax and an Income Tax

The prior part showed that investors could opt out of a simple tax by paying a fixed charge based only on the value of the asset, the holding period, and the interest rates. From the investor's perspective, opting out converts the income tax into a wealth tax. Some scholars have seized on this result to argue that income taxes are equivalent to wealth taxes.[36] This part cautions us when extending this seeming indifference to the government itself.

In his classic article on the taxation of risk, Professor Louis Kaplow identifies the criteria for deeming two systems of tax equivalent:

Two tax regimes are deemed equivalent if, for any choice of a tax

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[34] *See generally supra* part II.C.

[35] *See* Domar & Musgrave, *supra* note 3; *see also* Michael Livingston, *Risky Business: Economics, Culture and the Taxation of High-Risk Activities*, 48 TAX L. REV. 163 (1993).

[36] *See* Kaplow, *supra* note 2, at 791.

rate and a government portfolio under one regime, there exists a tax rate and a government portfolio under the other regime such that

(1) for any return that might be realized on the risky asset, investors have the same after-tax wealth in period 1 under both regimes;

(2) for any return that might be realized on the risky asset, the government has the same revenue in period 1 under both regimes; and

(3) total investment in each asset in period 0 is the same under both regimes.[37]

This test is stronger than saying that investors have the ability to opt out of income taxation. Instead, the results between a wealth tax and an income tax must be the same, or at least potentially the same, for all parties including the government.[38]

B. Risk Suppliers and Forward Contracts

Ophelia's opt-out transaction itself can be described in terms of risk. By taxing income, the government shares in Ophelia's gains and losses. The income tax transfers some risk to the government. Ophelia opts out by seeking additional risk. So far, we have assumed that she would make a leverage purchase of an additional ounce of gold. The problem, though, is that someone must supply Ophelia with the additional gold. This counterparty may or may not be subject to tax. We need to account for the counterparty's tax consequences if we want to analyze consequences to the government. By assuming that the counterparty sells actual gold, we complicate our inquiry. That sale may be taxable, and it produces proceeds that the counterparty may invest in a taxable fashion.

Rather than buying an ounce of gold today with leverage, Ophelia could enter into a forward contract to purchase an ounce of gold. A forward contract is a sales contract to be performed at a future time. The contract identifies the parties, the asset being sold, the future time for delivery, and the price for the asset (the "delivery price"). Typically, no money changes

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[37] *Id.*

[38] Professor Kaplow seems to accommodate potential equivalence by stating "there exists a tax rate and a government portfolio." *Id.*; *cf. infra* part V.C.3 (discussing government's entry into the market).

hands when the parties enter into the contract.[39] Instead, the delivery price is set so that it is initially "fair" to both parties. When the time for delivery comes, the selling party (the "short") delivers the asset and the purchasing party (the "long") pays the delivery price.

A fair delivery price is called the "forward price." Perhaps counterintuitively, the forward price mainly reflects the current price of gold and interest rates. It does not reflect expectations about the price of at the time of delivery. If gold is currently $1000 per ounce and interest rates are two percent per year, then the forward price for one year should be $1020. The explanation for this result is that a forward contract is effectively a leveraged purchase. If Ophelia borrows $1000 for a year to buy an ounce of gold, then in one year she will (still) own an ounce of gold but will need to repay $1020. She can possess gold by paying $1020 in one year. The forward contract gives her the same thing: ownership of gold in exchange for deferred payment. Thus, the forward price should equal $1020 in both cases.

A forward contract generally has no tax consequences until the parties settle their obligations.[40] If the parties settle with actual delivery of the underlying asset, they are treated just like any other parties to asset sale. Alternatively, the parties might settle with a cash payment. If Ophelia agrees to a forward price of $1020 but gold is actually $1500 in one year, she receives $480. If gold is actually $500 in one year, she pays $520.

We will assume that Ophelia agrees to a cash-settled forward contract on gold with a forward price of $1020. Her counterparty supplies her with

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[39] A party might demand collateral if worried about the other party's ability or willingness to perform in the future. But, the collateral itself can be invested in a wide range of liquid assets and is not an essential part of the forward contract.

[40] *See generally* DAVID H. SHAPIRO, TAXATION OF EQUITY DERIVATIVES ¶II.B.2.a (Tax Mgmt., U.S. Income Portfolios Library No. 188, 2000). The constructive-sale rules of I.R.C. § 1259 create a significant exception to his rule. If an investor owns an asset that has appreciated in value, then a forward contract to sell the asset in a later year will lock in the investors gain. Historically, though, the courts resisted attempts by the Service to treat the forward contract as a current sale of the asset. In the view of the courts, only an actual sale of the underlying asset triggers tax. *See id.* at II.B.2.a. n.98. Section 1259 overturns this approach by deeming the forward contract to be a constructive sale of the appreciated asset. *See id.* at II.B.2.c. If the investor has a *de*preciated position in the asset, however, the historical rules still apply, requiring an actual sale to realize the loss.

The constructive sale rules apply only where the investor holds an appreciated position in an asset and then takes an offsetting short position—i.e., selling the asset—in a forward contract or other derivative. A long position—i.e., buying more of the asset—does not justify taxation. In the examples of this article, investors use forward contracts to increase their exposure to assets they already own. Thus, the constructive-sale rules are not terribly relevant to our current analysis.

the additional risk she needs to offset the fifty percent tax on her gold. Consistent with the model developed so far, payments received are income and taxed at a rate of tax of fifty percent (or τ in general terms). Payments made under the contract are losses and receive symmetrical treatment.[41]

The forward contract is a convenient way to describe how a counterparty could supply additional risk to Ophelia. The contract gives Ophelia the same result as borrowing under the classic taxation of risk model. Moreover, it involves only one transaction (the contract) rather than two (a purchase and borrowing) and no necessary side effects (like her counterparty's investment decisions). Indeed, Ophelia can enter into this contract with anyone, whether or not that someone actually owns gold. Forward contracts also give us a plausible basis for thinking that parties could opt out without affecting the price of gold. Ophelia wants to buy more gold but so do other taxable gold investors wanting to opt out. The tax and resulting opt-out transactions appear to increase the demand for gold and other risky assets. Without a plausible substitute (like forward contracts), we would expect gold prices to go up.[42]

C. Analyzing Different Counterparties

1. A Similar Taxpayer Counterparty: Opt-Out Has No Effect on Fisc

Suppose Ophelia's risk supplier is another taxpayer also subject to a fifty percent tax. She agrees to a cash-settled forward contract to pay $1020 for one ounce of gold, settled in one year. Her risk supplier takes a short position on gold but takes no other actions (like buying gold) that could affect its taxes. From the government's perspective, nothing at all has happened, either when the parties enter the contract or when they settle. The forward contract is simply a side bet between Ophelia and the other taxpayer.[43] The government taxes one party's gains at fifty percent, but grants an offsetting deduction to the other party for its losses. In this scenario, the government is indifferent to the opt-out transaction. After netting its exposure to the forward contract, the government still taxes the gain or loss on Ophelia's gold.

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[41] If the underlying asset is capital in nature, then gains and losses should also be capital. *See* SHAPIRO, *supra* note 39, at II.B.3.a. Their source as foreign or domestic turns on the residence of the relevant taxpayer. *See id.* at II.B.4.

[42] *Cf. supra* part II.C.6 (assuming that the tax does not affect gold prices); *cf.* Kaplow, *supra* note 2 (same). Even with forward contracts, we might likely see an increase in prices. A third party *can* enter into a short contract with Ophelia even if the third party does not own gold. But, such third parties may strongly desire to hedge their risks by holding gold.

[43] *Cf. supra* part III (discussing side bets).

As before, Ophelia is indifferent between the fifty percent income tax and a wealth tax. The government is not. Under the income tax, the government bears fifty percent of the gain or loss from gold. This is the same result for the government had Ophelia not opted out in the first place.

2.   Tax-Exempt Counterparty: Opt-Out Amplifies Government's Exposure to Investment Risk

Assume now that Ophelia enters into a forward contract with a tax-exempt counterparty (like a foreigner or a charity). The counterparty faces no tax consequences on its side of the forward contract,[44] but Ophelia does. The opt-out transaction is no longer a wash from the government's perspective. Rather than converting the income tax into a wealth tax, though, the opt-out transaction *amplifies* the government's exposure to the price of gold. Again, we assume Ophelia enters a one-year forward contract to buy gold for $1020 but settling with cash.

- If gold goes up by fifty percent to $1500, Ophelia has a gain of $500 from the physical gold and $480 from the forward contract. Her tax-exempt counterparty has a $480 loss from the forward contract, but the government grants it no tax benefit. The government taxes the $500 of gain from Ophelia's physical gold and $480 from the forward contract. Had Ophelia not opted out, the government would tax only the $500 of gain.
- If gold goes down by fifty percent to $500, Ophelia has a loss of $500 from the gold and $520 from the forward contract. Her tax-exempt counterparty has a $520 gain from the forward contract, but it pays no taxes. On a net basis, the government grants a tax benefit to Ophelia based on the $500 from the physical gold and $520 from the forward contract. Had Ophelia not opted out, the government would grant a tax benefit based solely on $500 of loss.

In rough terms, the opt-out transaction has doubled the government's exposure to the risk of gold-price fluctuations. The outcomes to the government are clearly different from a tax on Ophelia's initial wealth.

For the sake of completeness, consider three other scenarios. First, Ophelia might find a counterparty that pays tax but at a lower rate. In that case, our result would be somewhere between that of this subsection (tax-exempt counterparty) and the prior (fully taxable counterparty). Second, Ophelia might find a higher-rate counterparty. The next subsection,

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[44] The foreigner, for example, would have foreign source income, and would not pay tax. *See* Livingston, *supra* note 34.

however, considers what happens when Ophelia transacts with the government itself, which we can think of as being a taxpayer subject to a 100% rate (i.e., all its gains and losses belong to the government).

3. The Government Itself: Opt-Out Converts Income Tax to Wealth Tax

The final case is when the government itself supplies the risk that Ophelia wants. If the government itself acts as counterparty to Ophelia, then it does indeed convert the income tax into a wealth tax.[45] Assume that Ophelia enters into a one-year forward contract to buy an ounce of gold from the government for $1020.

- If gold goes up by fifty percent to $1500, Ophelia has gain of $500 from the physical gold and $480 from the forward contract. The government collects tax from her equal to $490. The government has a loss under the forward contract equal to $480. On a net basis, the government collects $10.
- If gold goes down by fifty percent to $500, Ophelia has a loss of $500 from the gold and $520 from the forward contract. Ophelia has a total loss of $1020, and the government grants her a tax benefit equal to fifty percent of that, or $510. The government collects $520 from the forward contract. On a net basis, the government collects $10.

In both cases, the government collects a flat amount, equal to one percent of Ophelia's starting wealth. Thus, if—but only if—the government supplies Ophelia with the additional risk she demands, the government converts the simple income tax into a wealth tax.[46] The government could directly tax Ophelia's wealth without the bother of the forward contract.[47] The government could conversely convert a wealth tax into an income tax

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[45] David Weisbach and Louis Kaplow both acknowledge the role of government adjustments in the equivalence argument. Thus, they assume that the government will enter transactions that satisfy investor demands for additional risk.

[46] This case also has the attractive feature that Ophelia's counterparty—the government—has no obvious incentive to buy physical gold. We would expect no effect on prices. *See* Kaplow, *supra* note 2.

[47] In the United States, however, a federal wealth tax may be subject to constitutional challenge. In *Pollock v. Farmers' Loan & Trust Company*, 157 U.S. 429 (1895), *aff'd on reh'g*, 158 U.S. 601 (1895), the U.S. Supreme Court held that Congress had no constitutional authority to levy a property tax or to tax the income from property. The 16th Amendment, ratified eighteen years later, authorized Congress to tax income from property. U.S. CONST., amend. XVI. Whether *Pollock* would bar a federal tax on property or wealth today is debatable. *See* Calvin H. Johnson, *Purging Out* Pollock: *The Constitutionality of Federal Wealth or Sales Taxes*, 97 TAX NOTES 1723, 1734 (2002); Erik M. Jensen, *Interpreting the Sixteenth Amendment (by Way of the Direct-Tax Clauses)*, 21 CONST. COMMENT. 355 (2004).

with forward contracting.[48]

D. *Evaluating the Equivalence Interpretation*

Wealth and income taxes theoretically produce equivalent results as the government could enter the market and supply Ophelia and other taxpayers with the additional risk they want.[49] As a practical matter, we should question whether the government could enter the market to achieve actual equivalence. The government faces scrutiny when investing and may feel constrained to invest in enterprises that advance political rather than economic interests. Moreover, the government may not be able to reach every kind of investment. It can (and does) hold gold investments. It may not so easily be able to invest in every type of investment (like real estate or closely held businesses). Only by entering the market can the government convert an income tax into a wealth tax (or vice versa), and there may be significant frictions in doing so.

Even if the government could not effortlessly switch between wealth and income taxes, the equivalence interpretation still has power. We could draw an analogy by thinking of a private investor who is trying to decide whether to invest $1 million in risky assets (like stocks) or less risky assets (like bonds). Both portfolios have the same value, ex ante. One, though, has more risk and potentially a higher return. The investor would need to choose one that fits her personal needs the best. Similarly, a government might choose between two forms of taxation. A wealth tax would produce a low but stable source of revenue. An income tax would produce a higher, more volatile source of revenue. Both have equivalent values in a financial sense, but one might be better suited to funding government activities.

VI. THE SIMPLE TAX DOES NOT DISCRIMINATE AMONGST INVESTMENTS OF VARYING RISK

Even if wealth and income taxes are not equivalent, the taxation of risk model developed so far tells us something important about the simple income tax. Not only may Ophelia opt out of taxes on gold, but the size and cost of doing so do not depend on either the riskiness of gold or even its expected return. We should consider this a feature, not a bug. When choosing among assets (e.g., gold bullion or Google stock), the simple tax

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[48] The government would enter into (long) contracts to buy gold, exposing it to the risk of increase or decrease in the price of gold.

[49] Conversely, the government imposing a wealth tax does not capture the gain or loss from private investment. It could, however, enter the private market to obtain, rather than supply, risk. Doing so would effectively convert a wealth tax into an income tax.

gives Ophelia no reason to prefer one over another. From her ex-ante perspective, they have the same tax burden because the cost of the opt-out transaction does not depend on their risk or even expected returns. Thus, Ophelia's decision to invest in Google or gold is not distorted by the idealized-tax system we described above.

This result may seem counter to the idea that the income tax applies to outcomes. Risky investments not only lead to differing outcomes, but they should have higher expected returns as well.[50] Risk-free assets earn a low rate of return. Risky assets should, on an expected basis, earn an additional risk premium. The taxation-of-risk model suggests that the risk premium—the return from bearing risk—is not burdened with higher taxes.[51]

The explanation for this anomaly comes from examining the opt-out transactions, in particular the forward-contract version.[52] As described above, the "fair" price to pay for gold, delivered in one year, is the current price of gold plus interest. The fair price has nothing to do with the expected returns on gold. Thus, an investor (so long as she is creditworthy) can obtain the risk—and the risk premium—from gold simply by entering into a forward contract. And, as we saw in part III, the income tax does not burden side agreements like forward contracts.

This result is a manifestation of "risk-neutral valuation" that pervades the theory of derivatives.[53] When we value derivatives, we simply assume that the underlying asset grows at the risk-free rate of return. We disregard expected returns, risk premiums, risk preferences, and the like. As the leading text on derivatives states, "[r]isk-neutral valuation is a very important general result in the pricing of derivatives. It states that, when we assume the world is risk-neutral, we get the right price for a derivative in all worlds, not just in a risk-neutral one."[54]

Risk-neutral valuation extends directly to the income tax via the classic model described above. The simple tax, discussed above, is essentially a derivative contract between the government and the taxpayer. The government does not really own the taxpayer's assets, but has a claim that is derived from the asset's value and growth (or decline) over time. If the asset goes up, the government takes a share of the growth. If asset goes down, the government pays for some of the loss. According to risk-neutral valuation, we value the government's claim assuming that assets grow at the risk-free rate, a result completely consistent with the findings of this

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[50] *See* Zelenak, *supra* note 14, at 884–91.

[51] *See id.*

[52] *See supra* part V.B.

[53] *See* HULL, *supra* note 18, at 258.

[54] *Id.*

article and prior scholars.

Characterizing the income tax as a derivative contract allows us to interpret the classic taxation of risk model. According to risk-neutral pricing, investors can freely obtain risk premiums without investing any cash but by investing in derivatives. Our usual categories of capital and labor do not seem to apply. The return on the derivatives contract is clearly not a return from labor, but it is not a return on capital either because it does not require capital at all. Our categories fail, and we see a form of income that seems to go untaxed, at least from the perspective of private parties.

As we will see below, characterizing the income tax as a derivatives contract gives us an enormously powerful tool. Until now, we have been analyzing a simple tax that has normative appeal but no real-world application. Calling the income tax a derivatives contract allows us to move from the simple, classic taxation-of-risk model to more sophisticated models that account for strategic behavior, responses to such behavior, and alternative models of income taxation.

VII. TIMING OPTIONS CAUSE THE INCOME TAX TO FAVOR RISK

A. Timing Options and Loss Harvesting

We now begin to relax the assumptions specified in part II.C. In particular, we will revisit the idea of a fixed holding period and a tax imposed at its end. We continue to assume that the taxpayer still has an *economic* holding period. At its end, the taxpayer will want to liquidate the investment in order to fund consumption or another investment opportunity. During the interim, the asset will either go up or down in value. If it goes up in value during the interim, the taxpayer does nothing. If it goes down in value, however, the taxpayer strategically capitalizes on the drop. She sells the asset, triggers a taxable loss, and reinvests the proceeds immediately in the same asset.[55] Absent transaction costs and legal restrictions, the investor has no reason not to trigger an immediate tax loss. This strategy, known as "loss harvesting" exploits the "timing option" that the taxpayer holds when deciding when to trigger gains and losses.

Commentators have previously noted the "timing option" and "loss harvesting" opportunities from unfettered freedom to sell.[56] U.S. tax law combats loss harvesting by deferring loss deductions in certain situations.[57]

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[55] In the United States, this strategy would not work for securities under the wash-sale rules. *See* I.R.C. §1091.

[56] *See* George M. Constantinides, *Capital Market Equilibrium with Personal Tax*, 51 ECONOMETRICA 611 (1983).

[57] *See, e.g.,* I.R.C. §§ 1091, 1211, & 1212.

Allowing successful investors to recognize loss on losers while deferring gain on the winners would threaten the fisc. However, the government could respond simply by raising rates, although doing so could exacerbate "lock in"—the phenomenon in which a taxpayer continues holding suboptimal investments in order to avoid tax on previously accrued gains.

This part uses the taxation of risk model to identify another reason why higher rates could not combat loss harvesting. The prior part argued that the simple tax, developed earlier, is beneficially neutral with respect to risk. Loss harvesting, however, threatens this neutrality because the timing option is most valuable with respect to risky assets.

B. *Refinancing the Opt-Out Transaction*

Recall that the basic model has the investor borrow to finance an additional purchase of the asset. If the investor starts with an asset worth S_0, she must borrow $\tau / (1 - \tau) S_0$ in order to fund the additional purchase. In this part, it will be convenient to assume that the investor prepays her interest on this loan every year using a rate of r^*. Without harvesting, prepaid annual interest comes from (10) by assuming $T = 1$. Annual prepaid interest equals

$$(11) \qquad S_0 r_* \text{where } r_* = (1 - e^{-r})\frac{\tau}{1 - \tau}$$

Our investor will harvest losses at some intermediate time $= j$, if the price at that time, S_j, falls below S_0. Loss harvesting is simply a sale at the price of S_j, generating a taxable loss.

The classic model establishes that the opt-out transaction fully hedges against tax consequences at any time. By selling at a loss, the investor gets a tax benefit (a deduction) but must repay the principal on the loan she took out with the initial opt-out transaction. The tax benefit and the loan repayment cancel. The investor is left with S_j, just as we would expect given the design of the opt-out transaction. What, then, is the benefit of loss harvesting under the taxation-of-risk model?

She still plans to hold the asset until some later time $= T$. The benefit of harvesting comes when the investor reestablishes the new opt-out transaction. Now she pays a lower amount of annual prepaid interest equal to $S_j r_*$. In other words, the investor benefits with a lower annual cost on her opt-out transactions after harvesting. Whenever the asset reaches a new minimum price, the investor harvests the losses and forces ongoing opt-out cost again.

Effectively, the timing option allows the investor to "look back" over the asset performance since time $= 0$ to find the minimum price, S_{min}. This

minimum price, not the initial price, establishes the cost of financing the opt-out transaction for future periods. Thus, after the initial year (time = 0), the annual cost of the opt-out transaction is

(12) $S_{min}\, r_*$

The best (tax) case scenario for a loss harvester would be a temporary dip in the asset price to some small amount. Suppose Ophelia buys gold worth $1000. To opt out of a twenty percent tax, she must fund a $250 purchase of gold. Gold momentarily dips to $1 an ounce. She immediately sells and reestablishes her opt-out transaction. Now, however, she needs to fund a mere twenty-five cent purchase. Gold immediately goes back to its prior price of $1000 per ounce. Ophelia has opted out of the twenty percent tax, but her cost is based on financing the twenty-five cent purchase.

C. Comparing the Refinanced Cost to the Simple Model

Let us take the simple model as our baseline, the result that we ideally would see under a realization-based tax. According to this baseline, however, the cost of the opt-out transaction should be based on the initial price, S_0, as described above in (11). Loss harvesting gives the investor a lower cost, based on S_{min}, described in (12). The annual benefit from loss-harvesting is the difference between these two amounts.

(13) $[(S]_0 - S_{min})r_*$

This benefit has the same form as a so-called "look back" option.[58] Identifying this form allows us to use an option-pricing model to value the benefit from loss harvesting.

D. Relationship Between the Cost of Opting Out and Risk

Let us suppose that Ophelia has a true holding period of ten years for an investment of $1000. She faces a twenty percent tax and can obtain funds for an after-tax cost of two percent. Under the simple model developed above, her prospective cost of opting out should be about $45.[59] This cost does not depend on the investment that Ophelia chooses because the simple tax does not discriminate based on risk.

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[58] To be more precise, the payoff is that of a fixed look-back put option. The strike price is the initial price, S_0. *See* HULL, *supra* note 18, at 582–84.

[59] *See supra* part IV.C.

With loss harvesting, she can lower this cost. The amount by which she lowers her cost, however, depends on how much investment risk she undertakes. Risk, in financial terms, is simply volatility, a measure of uncertainty regarding an asset's future value.[60] A more volatile asset gives Ophelia a better chance at a dip in price, allowing her to reestablish the opt-out transaction at a low price. Of course, Ophelia would suffer from a dip, but the higher volatility gives her more economic upside potential as well.

The prior section described loss harvesting as a look-back option, letting the investor look back to a prior minimum price when setting the basic opt-out transaction. The following graph shows how these look-back options lower the cost of the opt-out transaction as investment risk increases from 0 to 1.0.[61] The top horizontal line represents the cost of the opt-out transaction without harvesting, a flat line of about $45. The lower curve represents the expected cost with harvesting, based on the pricing model for look-back options.
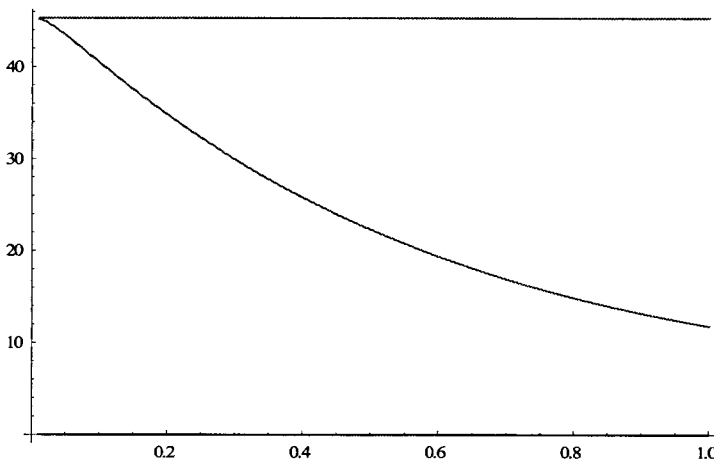
The graph shows a steep decline in the expected cost of the opt-out transaction as she takes on more risk.[62] Thus, loss harvesting not only lowers government revenue. It distorts investment choices, causing

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[60] *See* HULL, *supra* note 18, at 811.

[61] Investment risk here is measured by standard deviations. Risk of "1.0" is 100% standard deviation.

[62] The graph relies on some assumptions about the frequency of loss harvesting and the frequency of resetting the opt-out transaction. In particular, I assume that the investor can always lock in a lower price at any time the stock falls below a prior minimum. But, the investor must wait to lower her annual cost from the opt-out transaction until the start of the next period. So, the cost of the opt-out transaction for the first year (from time = 0 to time = 1) is fixed and cannot be changed. The cost of the opt-out transaction for the second year (from time = 1 to time = 2) is based on the minimum price of the asset over the first year.

investors to prefer more risky assets.

VIII. LOSS LIMITATIONS COMBAT HARVESTING BUT CAUSE THE INCOME TAX TO DISFAVOR RISK

A. Loss Limitations

U.S. tax law responds to loss harvesting by limiting the ability of investors to deduct losses. Investors can always deduct capital losses against capital gains. But, they cannot always use capital losses to reduce other income. Under I.R.C. § 1211, individuals can deduct only $3000 of losses that remain after netting against gains. After the $3000 deduction, remaining losses carry forward to later taxable years indefinitely until used.[63] Corporations do not enjoy the $3000 deduction and can carry excess losses forward for only five years.[64]

U.S. law also contains a set of "wash-sale rules" that disallows a loss on stock or securities if the investor acquires "substantially identical stock or securities" thirty days before or after the day the loss was harvested. So, the investor could replace the investment thirty-one days after loss harvesting, going thirty days without the investment, or the investor could preemptively replace the investment thirty-one days before harvesting, going thirty days with double the investment.[65] Conceptually, U.S. law disrupts the investor's preferred asset allocation as a price of harvesting the loss.

Capturing the effects of these loss limitations is difficult. The wash-sale rules disrupt the investor's investment strategy, but it is difficult to put a price on this disruption. As for the limits of section 1211, their effect depends on the investor's overall portfolio and whether it produced gains in a particular year. Some investors can use losses, but some cannot.

B. Modeling Complete Disallowance

The prior part modeled extreme loss harvesting. Whenever the asset price hit a new low, the taxpayer could lower the cost of the opt-out transaction. This section does something similar with loss limitations, assuming that losses produce no tax benefit at all. Thus, the government effectively holds an option on the investor's asset. If the asset goes up in

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[63] *See* I.R.C. § 1212(b).

[64] *See* I.R.C. § 1212(a)(1)(B). Corporations do have one advantage over individuals, though, as they can carry loss *back* to the three prior taxable years. *See* I.R.C. § 1212(a)(1)(A).

[65] *See* I.R.C. § 1091.

value, the government taxes the gain. If the asset goes down in value, the government bears none of the loss.

The investor could try to opt out of this harsher tax by buying her own option in the private market. The option would offset the tax but would itself subject the investor to tax if the asset grows enough in value. The investor could buy a second option to cover tax on the first option. In theory, this process would go on infinitely, but three or four options will probably suffice for practical purposes.[66]

The following graph shows how the opt-out transaction increases in cost as the risk of the investment rises. As before, we assume a ten year holding period, a twenty percent tax, a two percent after-tax interest rate, and a $1000 initial investment. The simple model yields a flat cost of $45, regardless of risk. Now, the cost increases dramatically. Even at relatively moderate levels of volatility like 0.25, the cost more than doubles.


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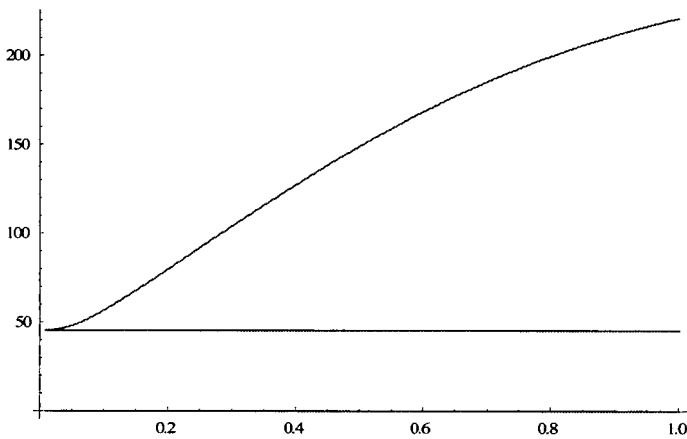
[66] A detailed example follows:

Suppose the investor has $1000, buys an asset with volatility/risk of 30%, expects to hold it for 10 years, faces a 20% tax rate on gains, and has the ability to borrow at an after-tax rate of 2%. As we have seen before, the simple model points to an opt-out cost of about $45. The cost is much higher with complete loss disallowance. Option-pricing models assume that assets grow at the risk-free rate. After-tax, we would expect a growth rate of 2%. The $1000 should grow to about $1221 after tax with continuous compounding. A before-tax growth rate of 2.44% produces the same result, provided the tax applies at realization.

To hedge against the tax on her asset, the investor buys a call option on the asset that pays if the asset increases in value. Using the assumptions so far, the option price is about $443, but the investor needs only 20% of this option for a cost of about $89. If the asset grows more than $443 dollars, however, the first option produces gain. So, the investor must buy a second option with a strike price of $1443. This second option price is about $326, but the investor only needs 4% of this option (20% of the first option which covers 20% of the asset). The cost of the second option is about $13. So far, the overall cost is about $102. We could continue opting out indefinitely, although successive options become less and less significant. The third option would cost about $2, and the fourth would cost less than 50 cents. We can safely say that the cost of opting out is pretty close to $104.

C. Interpreting the Model

The graph overstates the impact of loss limitations as it assumes no tax benefit for losses. U.S. tax law, however, forces investors to match their capital losses against capital gains. Thus, losses are deferred until the investor has a matching capital gain. Still, deferral does reduce the value of losses. Some investors may never be able to use their losses, particularly if the overall market declines. Loss limitations inevitably burden some investors, even if the model overstates the burden.

Moreover, the model shows that policymakers could be overzealous in combating loss harvesting, the primary reason for limiting losses in the first place. The following graph combines the prior two, suggesting that the burdens from complete loss disallowance exceed the benefits from extreme loss harvesting. Again, we have an investor with $1000 facing a twenty percent tax, a ten year holding period, and two percent after-tax interest rate. The horizontal line is the flat $45 cost of opting out of the simple model. The curve below represents unrestrained loss harvesting, showing the cost diminishes as risk increases. The curve above represents complete loss denial, showing that the cost increases as risk increases.
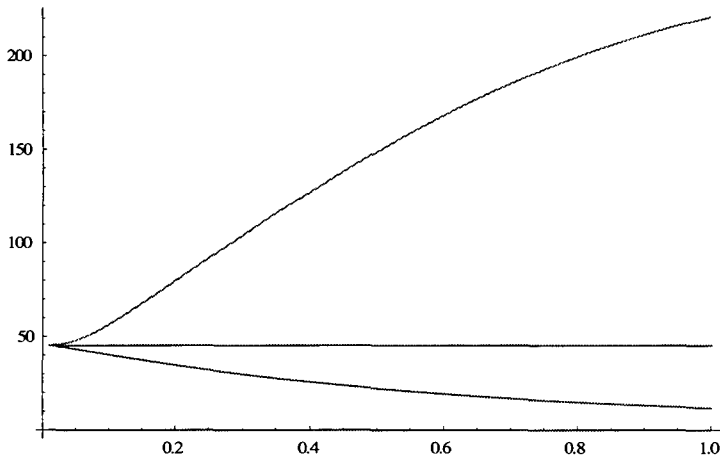
Earlier, this article argued that the simple tax does not discriminate based on risk, a normatively desirable feature. We see now how tax-motivated selling (loss harvesting) encourages risk taking, but policy responses (loss limitations) discourage risk taking. The policy goal should be to discourage loss harvesting with the least possible distortion on risk taking. Policymakers could "solve" the problem if they abandon taxing capital altogether by shifting to a consumption tax. Another extreme solution would be to tax capital appreciation or depreciation on a year-to-year basis, even if not recognized. The next part shows how the taxation of

risk model extends to such an accretion tax.

IX. THE MODEL EXTENDS TO ACCRETION (OR MARKED-TO-MARKET) SYSTEMS OF TAXATION

A. *The Allure of Accretion Taxation*

An accretion tax would tax gains and recognize losses according to movements in asset prices. In other words, the tax would be imposed on a "marked-to-market" basis. This method finds favor among tax law academics[67] and applies to a few types of financial instruments under U.S. law, like futures contracts.[68] Several scholars have proposed regimes that would tax gains, especially those on publicly traded assets, on a "marked-to-market" basis.[69] Supporters justify these regimes by showing they achieve equity between returns on capital and income. Capital appreciation gives the holder power and wealth, intangible benefits that are themselves much like consumption.[70]

In this part, I extend the classic taxation-of-risk model to accretion taxes. Two results will stand out. First, accretion taxes are neutral as to risk, just like the simple tax developed above. Investors who wish to opt out of the accretion tax face a significant constraint, however, as they must pre-commit to some specific holding period. Second, the model may undermine some of the fairness rationales that support accretion tax. Investors could convert an accretion tax into a fixed charge on starting wealth, just as they can with a realization based tax. Thus, the accretion tax may not burden investors who achieve greater wealth and power over their holding periods.

B. *Opting Out of an Accretion Tax*

1. The Goal of Opting Out

Remember Ophelia and her ounce of gold, currently worth $1000.[71] Opting out of a realization tax was fairly straightforward, and we could even be a bit casual about how she goes about it. She would borrow funds to buy more gold, and she could borrow without any set time period. She

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[67] *See generally* BORIS I. BITTKER & LAWRENCE LOKKEN, FEDERAL TAXATION OF INCOME, ESTATES AND GIFTS ¶ 3.5.2 (2012).

[68] *See* I.R.C. § 1256.

[69] *See* BITTKER & LOKKEN, *supra* note 66, ¶ 3.5.2 n.4.

[70] *See* Jeff Strnad, *Periodicity and Accretion Taxation: Norms and Implementation*, 99 YALE L.J. 1817 (1990).

[71] *See supra* part III.

might prepay interest, pay interest periodically, or pay at the end of her holding period. Opting out of the accretion tax is technically more difficult. Our goal, as before, is to offset the tax, but now the tax is imposed repeatedly over the holding period.

Under this accretion tax, we will need to be more precise in the method and goals of opting out. In a tax-free world, Ophelia would start with one ounce of gold and end with one ounce of gold. Let us define her goal as having wealth worth one ounce of gold at the end of some holding period, after paying all taxes. Ophelia simply needs to know how much gold to start with. If she had a one-year holding period, the answer is supplied already. With an after-tax interest rate of two percent and a tax rate of twenty percent, Ophelia must start with about 1.005 ounces of gold using (10) If gold costs $1000 per ounce, she would have to pay $1005 for an ounce of gold, after tax, in one year.

2.   Opting Out of Multiple Years

We know how to opt out of a one-year tax—buy 1.005 ounces of gold. Extending the opt out to several years means that we simply work backwards. Suppose Ophelia has a two-year holding period:

- Year 2: Ophelia wants 1 ounce of gold at this time.
- Year 1: At this time, Ophelia wants to be in a position to hold 1 ounce of gold in one year. So, Ophelia needs 1.005 ounces of gold to produce 1 ounce of gold in the course of a year.
- Year 0 (today): Ophelia wants to be in a position to hold 1.005 ounces of gold in one year. To achieve this goal, she needs about 1.010 ounces of gold.

We can generalize this result for any interest rate, tax rate, and holding period. Our taxpayer defines a unit of wealth at some time in the future, T years from now. The unit might be anything—an ounce of gold, 312 shares of Google common stock, or a balanced portfolio of stock and equity. Let us define that unit as S_T. The question is how many units does she need today to produce one unit in the future. Since we know the price today is S_0, we can convert the units required today into monetary terms.

The details of this work are left to the Appendix. We express the interest as a pre-tax rate, R. Assuming that taxes apply continuously (i.e., whenever the asset moves in price), this leads to a more "elegant" result that also relates to prior work on retrospective taxes.[72] The model easily handles annual taxes.[73] By properly opting out, the investor starts with the amount

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[72]   *See infra* part IX.B.3.

[73]   *Cf.* Strnad, *supra* note 69 (arguing that continuous-time taxation is the theoretical

on the left of the arrow and ends with the after-tax amount on the right, her identified goal.

(14) $$e^{RT\tau}S_0 \to S_T$$

We could say that the prospective cost of opting out for T years is given by

(15) $$S_0[(e]^{RT\tau} - 1)$$

Following the financial crisis, elegant models smell of fiction if not danger.[74] Yet unlike the financial models of Wall Street, the relationship given above does not turn on any assumptions about how asset returns are distributed. The Black-Scholes model of option-pricing assumes that prices follow a normal or Gaussian distribution,[75] but the relationship above is completely agnostic as to the movement of asset prices.

More fundamentally, the relationship is not even offered to represent our actual system of taxation. It clearly does not. Instead, it represents the theoretical ideal of Haig-Simons taxation, at least under a flat rate of tax. The assumption of continuous-time taxation significantly deviates from the Haig-Simons expectation of annual taxation. As Professor Jeff Strnad notes, there is no normative reason for the "periodicity" of taxes to be determined by the Earth's orbit around the Sun.[76] Haig-Simons points to taxation on a more frequent basis than we see under a realization standard. If annual taxes are better than realization taxes, then monthly taxes must be better still. Theoretically, then, continuous taxes—applied instantly when prices move—are best of all.

3.   Retrospective Cost and the Auerbach Tax

We can equivalently view the accretion tax as causing an asset to experience a fixed rate of decay in value over time. An asset worth S_0 today will be worth something less than S_T at time $= T$. According to (14), the asset takes the following path:

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ideal).

[74] *Cf. e.g.*, Felix Salmon, *Recipe for Disaster: The Formula That Killed Wall Street*, WIRED MAG.(Feb. 23, 2009), http://www.wired.com/techbiz/it/magazine/17-03/ wp_quant (criticizing the "Gaussian copula" model used to model credit risk).

[75] *Cf.* NASSIM NICHOLAS TALEB, THE BLACK SWAN (2008) (criticizing use of Gaussian assumption in finance).

[76] *See* Strnad, *supra* note 69.

$$(16) \qquad S_0 = e^{RT\tau}\left(e^{-RT\tau}S_0\right) \rightarrow_\backslash e^{-RT\tau}S_T$$

Along the same lines, we could value the retrospective cost of opting out of the accretion tax. At time = T, the investor has the amount described in (16). Absent taxes she would have S_T. The difference between the two is the retrospective cost opting out:

$$(17) \qquad S_T - e^{-RT\tau}S_T = \left(1 - e^{-RT\tau}\right)S_T$$

More than twenty years ago, economist Alan J. Auerbach proposed a "retrospective capital gains" tax that, while levied only upon realization, is nevertheless financially equivalent to accretion taxation.[77] According to Auerbach, his tax offers the best of all worlds. It avoids the advantages of deferral and the problems of "lock in" associated with realization-based taxes. Because the Auerbach tax is imposed on realization, it avoids the liquidity and valuation difficulties inherent in taxing assets on an annual basis.

Computationally, the Auerbach tax is exactly the same as the retrospective cost in (17).[78] Suppose that taxes are twenty percent, the holding period is ten years, and the before-tax interest rate is 2.5%. An investor who sells an asset worth $1000 at the end of 10 years would then pay tax of about $49. The Auerbach tax is financially equivalent to an accretion tax, at least in the sense that a wealth tax is equivalent to an income tax.[79]

Note that the Auerbach tax clashes with what might be called "tax system aesthetics."[80] The Auerbach tax does not depend on the amount of gain and applies even if there is a loss. If the investor bought for $2000 but sold for $1000, she would still pay the Auerbach tax. The tax depends on (1) the price received for the asset (or the "amount realized" under the U.S. tax code), (2) the interest rate, (3) the holding period, and (4) the tax rate. What is interesting is that the Auerbach tax is functionally equivalent to accretion taxation under the extended taxation of risk model.

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[77] *See* Alan J. Auerbach, *Retrospective Capital Gains Taxation*, 81 AM. ECON. REV. 167 (1991).

[78] *See id.* at 170.

[79] *But cf. supra* part V (discussing equivalence between income and wealth taxes).

[80] *See* Jeff Strnad, *Taxing New Financial Products: A Conceptual Framework*, 46 STAN. L. REV. 569, 592 (1994) (referring to "tax system aesthetics").

C. *Essential Differences Between Realization and Accretion Taxation*

1. Holding Periods

In order to opt-out of the accretion tax, an investor must pre-commit to a specific holding period. A transaction that works for ten years would not work for five years or fifteen years. This is not to say that the investor has any legal duty to hold for the pre-committed period. It is to say, though, that a particular opt-out transaction works only for a particular holding period.

For example, suppose an investor faces a twenty percent tax, a two percent after-tax interest rate (2.5% before tax), and plans for a ten-year holding period. Her initial holding of the asset, S_0, must be increase to about $1.05127 \times S_0$. At time = 10, she would be assured of having S_{10}. If she wanted to continue holding the asset, she certainly could. She could even "opt out" of taxes again, but her cost would be based on S_{10}, not on S_0. If she held until time = 5, she would have $1.02532 \times S_5$. From her perspective, she has devoted "too much" of her initial wealth to the asset. A smaller initial investment would have produced S_5.

In contrast, the simple tax modeled at the beginning of this article had flexible holding periods. The investor could gross up at time = 0 and pay a flat annual interest charge to do so. The investor could abandon the transaction at any time = j and be assured of having exactly S_j. Indeed, the holding period of the simple tax is too flexible as it allows investors to engage in loss harvesting.

This idea is related to the timing options discussed earlier. Under a realization based tax, the investor chooses when to face the tax consequences of gain or loss. She can be flexible, accelerating loss or deferring gain as she likes with a flexible holding period. The accretion tax removes this flexibility. To opt out, the investor must identify a holding period ahead of time.

2. Magnitude of Burdens

The potential burdens of accretion taxes are greater than those of the simple tax. The prospective cost of opting out of the simple tax is[81]

$$(18) \qquad\qquad S_0(1 - e^{-rT})\frac{\tau}{1 - \tau}$$

The comparable cost for the accretion tax is[82]

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[81] *See* Equation (10).

[82] *See* Equation (15)0.

(19) $$S_0 \llbracket (e]^{RT\tau} - 1)$$

For short holding periods, the differences may not seem too great. Suppose that the after-tax interest rate, r, is two percent, and the tax rate is twenty percent. The before-tax interest rate, R, is 2.5%. Our investor holds an asset worth $1000. To opt out of the accretion tax costs about $51. To opt out of the realization-based tax costs about $45.

Over longer periods, however, the differences become much greater. Under the realization based tax, the investor is sure that she needs no more than $250 for a permanent exemption from a twenty percent tax. That is the maximum cost of the opt out as T or r grows. In contrast, the cost for the accretion tax has no maximum amount. The cost grows without bounds as holding periods and interest rates increase. A supporter of limited government might find the realization-based tax appealing because it caps the government's share of wealth in a way that the accretion tax fails to do.

The following chart compares the prospective cost of opting out of the two different taxes. As before, the investor has $1000 to invest and faces a twenty percent tax and a two percent after-tax interest rate. The vertical is prospective cost. The horizontal is holding period.
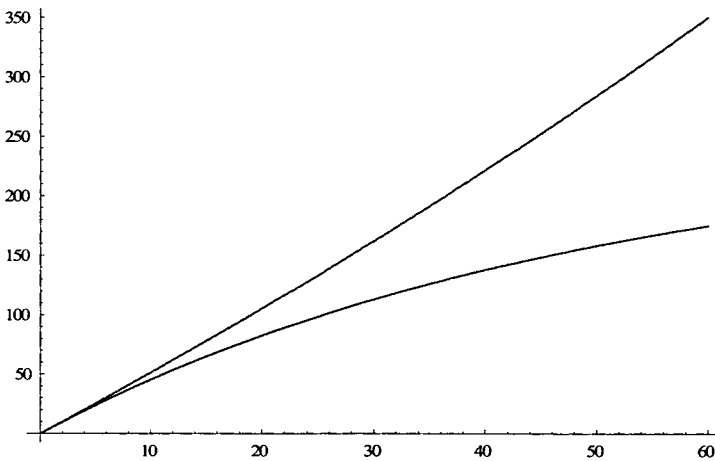
We might interpret these two curves as representing acceptable boundaries for taxing capital income. The top curve represents an instantaneous accretion tax, applied whenever the investment moves in value. The bottom curve represents realization, applied whenever an asset is sold but not before. What is interesting is that, for even moderately long holding periods like ten years, the choice does not appear consequential. Only for long holding periods, such as twenty or more years, do the two approaches begin to diverge dramatically.

D. Fairness and Accretion Taxation

The accretion tax has obvious efficiency advantages over the realization-based tax. The ex-ante cost of opting out of the accretion tax is fixed. In contrast, realization-based taxes are plagued by timing options or loss limitations that distort investors' choices. Moreover, many assets produce annual yields, such as dividends and interest. All returns on property are taxed annually under accretion. In contrast, even under realization-based taxes, annual yields are taxed annually, often at higher rates.

Many tax scholars argue that accretion taxation is fairer than realization taxation. Accretion taxes put capital and employment income on an equal footing, taxing both annually. Several scholars, however, argue that the government should not tax returns on capital at all, taxing instead consumption. Professor Jeff Strnad links accretion taxation to consumption-tax ideals. He argues that increases in wealth give their holders power, prestige, and enjoyment. Wealth is, in effect, a form of consumption, even if it is not spent.

Professor Strnad argues that only accretion taxes effectively reach these changes in wealth. Indeed, the government has no principled reason to tax wealth changes only annually. It could tax them more frequently, perhaps even continuously. This part models an accretion tax that does just that. Nevertheless, it shows that investors can still opt out of this regime,[83] converting the accretion tax into an up-front fixed payment. Accretion taxes may not have fairness advantages over realization taxes.

X. CONCLUSION

The classic taxation-of-risk model might not accomplish what past scholars have suggested. Unless the government actively facilitates the conversion, income taxes and wealth taxes are inherently different from the perspective of the fisc. From the perspective of private parties, though, they might be equivalent. For this reason, we can say that a simple tax does not distort investment choices between various classes of risky assets.

The model, at its core, describes the income tax .as a derivatives contract. The government has a claim against taxpayers that "derives" from the value of their holdings. This observation opens up the income tax to

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[83] Professor Strnad, while praising the cleverness of the Auerbach tax, finds that it does not satisfy his ideals of taxing changes in wealth. According to Professor Strnad, an accretion tax, applied instantly to changes in wealth, satisfies his ideals. This part shows that the Auerbach tax and the accretion tax, applied in continuous time, are functionally equivalent, at least from the perspective of investors.

powerful analytical tools from option pricing models. We can now account for strategic behavior (timing options) and government responses to strategic behavior (loss limitations). In both cases, we see distortions from the simple model. Timing options distort in favor of seeking risk, while loss limitations distort in favor of avoiding risk.

Finally, the model can extend beyond the simple realization based tax to proposals for marked-to-market or accretion taxation. Investors can effectively opt out of those taxes if they know their holding period ahead of time. This observation gives us two results. First, we can prospectively value accretion taxes and compare them to realization taxes. Second, we should question whether accretion taxes effectively reach annual (or more frequent) changes in asset values, as proponents suggest.

APPENDIX

I. INTRODUCTION

This Appendix formalizes the model for accretion taxes. As throughout the article, S_j is the value of some asset at time $= j$, T is the holding period, τ is the tax rate, and r is the after-tax cost of funds (either from borrowing or liquidating cash). We will generally assume borrowing. It is convenient also to use the before-tax cost of funds, $R = r / (1 - \tau)$.

II. SINGLE PERIOD OPT OUT

This section describes the simple-tax model with a view toward applying the model to the accretion tax. It is slightly different from the presentation in part IV as it begins with annual interest rates. Doing so allows the next section to shift to continuous time.

Today (time $= 0$) an investor plans on having an asset worth S_1 in one year (time $= 1$) after tax. This is the result in a tax-free world. In a taxable world the investor achieves this goal by both increasing in her initial investment with an out of pocket contribution and borrowing.[84] According to (4)

$$(20) \qquad \frac{S_0}{1-\tau} \rightarrow S_1 + \frac{\tau}{1-\tau} S_0$$

The right side of the arrow shows the investor's goal of S_1 plus an additional term, $\tau / (1 - \tau) S_0$. To drop the additional term, assume the investor borrows funds so that she repays $\tau / (1 - \tau) S_0$ at time $= 1$. This

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[84] *See supra* part IV.

repayment leaves her with S_1 at time = 1, the stated goal. The initial borrowing that leads to this repayment comes from dividing the repayment by $(1 + r)$ or by $(1 + R(1 - \tau))$ if we use before-tax rates. So, the initial out-of-pocket investment equals the grossed-up holdings(the left side of the arrow in (20)) less the initial borrowing. We have the following single-period path leading to S_1:

$$(21) \qquad \frac{S_0}{1 - \tau} - \frac{\tau}{(1 - \tau)(1 + R(1 - \tau))} S_0 \rightarrow S_1$$

Simplifying the left side leads to the following:

$$(22) \qquad \frac{1 + R}{1 + R(1 - \tau)} S_0 \rightarrow S_1$$

III. MULTIPLE PERIOD OPT OUT

Unlike the simple realization-based tax, an accretion tax applies periodically. The investor cannot opt out of taxation by extending her borrowings over multiple periods. To opt out of two periods of accretion, the investor applies the process of the prior section repeatedly. A two period opt out transaction leads to the following two steps.

$$(23) \qquad \left(\frac{1 + R}{1 + R(1 - \tau)}\right)^2 S_0 \rightarrow \frac{1 + R}{1 + R(1 - \tau)} S_1 \rightarrow S_2$$

Generalizing to T periods yields the following.

$$(24) \qquad \left(\frac{1 + R}{1 + R(1 - \tau)}\right)^T S_0 \rightarrow S_T$$

The path in (24) describes how an investor would opt out of an accretion tax imposed annually. Notice how we simply determine a factor that is multiplied against S_0.

IV. CONTINUOUS TIME

Financial economics often calculates interest using continuous compounding. Taxes, in contrast, apply annually. The justification for doing so is administrative. Normatively, annual accounting leads to small anomalies. Gain on January 1 is not taxed for more than a year. Gain on December 31 is taxed within a few months. Ideally, an accretion tax would

apply instantly whenever an asset went up or down in value. Following standard convention, we divide each year into n sub-periods. As the sub-periods become arbitrarily small, we see how the factor multiplied against S_0 changes.

$$(25) \qquad \lim_{n \to \infty} \left[\left(\frac{1 + \frac{R}{n}}{1 + \left(R/n\right)(1 - \tau)} \right)^{Tn} \right] = e^{RT\tau}$$

Based on (25), we can specify the initial investment required to achieve final wealth of S_T under continuous accretion taxation.

$$(26) \qquad S_0 e^{RT\tau} \to S_T$$

Alternatively, we can show how an initial asset S_0 changes over a period of time under continuous accretion taxation.

$$(27) \qquad S_0 \to S_T e^{-RT\tau}$$